Energy Approach to Earthquake-Induced Slope Failures and Its Implications

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Abstract: So far, earthquake-induced slope instability has been evaluated by force equilibrium of soil mass in engineering practice, which cannot evaluate failure deformation once large failure occurs. An energy approach is proposed here, in which the amount of earthquake energy is evaluated in conjunction with the gravitational potential energy dissipated in slope displacement including large flow deformations. Shake table tests of dry sand slopes are carried out in which the earthquake energy used for slope failure can be successfully quantified. Measured slope displacement can be reliably evaluated by the proposed energy approach based on a rigid block model if an appropriate friction coefficient of the slope is specified. The energy approach is then applied to hypothetical slopes, indicating that even if extremely large earthquake energy is considered, slope failures with long run-out distance will not occur unless friction coefficients reduce near to or smaller than slope inclinations.

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Introduction

Seismically induced slope failures have normally been evaluated based on equilibrium of forces acting on a potentially sliding soil mass. This force approach can evaluate a safety factor against slope failure, but cannot predict slide deformation, once failure occurs. From a viewpoint of the performance-based design or the risk evaluation of slope failures, it is very important to know not only the safety factor but also how large deformation develops and how far failed soil mass reaches down slope. The Newmark method (Newmark 1965) or its modifications by using finite-element method (FEM) analyses (e.g., Makdisi and Seed 1978) can evaluate displacement of a rigid soil block along a fixed slip surface based on a double integration of acceleration acting on it. In actual slope failures, however, sliding soil may not necessarily behave as a rigid body but deforms continuously without fixed slip surfaces. It sometimes tends to become destructive due to a shift from slow rigid-block slide to fast debris flow because the friction coefficient decreases drastically after the initiation of failure.

In order to evaluate slope failures including flow failures from their initiation to termination, an energy approach was proposed by Kokusho and Kanasawa (2003) and further developed by Kokusho and Ishizawa (2006). In that method, four energies: potential energy change by gravity $-\delta E_p$, earthquake energy contributing to slope failure $E_{EQ}$, dissipated energy in a sliding soil mass $E_{DP}$, and kinetic energy $E_k$ of the sliding soil mass, are correlated by the following equation:

$$-\delta E_p + E_{EQ} = E_{DP} + E_k$$  \hspace{1cm} (1)

or in an incremental form as

$$-\Delta E_p + \Delta E_{EQ} = \Delta E_{DP} + \Delta E_k$$  \hspace{1cm} (2)

Note that the potential energy change before and after failure $\delta E_p$ in Eq. (1) or $\Delta E_p$ in Eq. (2) is normally negative.

Once failure starts, the amount of the dissipated energy is critical to decide if it develops as a flow-type failure and how far it flows. In some time increments when earthquake shaking has already ended ($\Delta E_{EQ}=0$), if $\Delta E_{DP}$ is smaller than $-\delta E_p$, then it is clear from Eq. (2) that $\Delta E_k$ increases and the soil mass accelerates. It may also be inferred from Eq. (2) that a shift from slow slide to fast flow may occur not only due to an increase in $-\delta E_p$ but also due to a drastic decrease of $\Delta E_{DP}$ caused by pore-pressure buildup in liquefiable soil, strength loss in high-sensitivity clay, etc. In fast flow failures, soil mass will keep flowing unless the kinetic energy plus the subsequent potential energy change are all dissipated in the sliding soil mass. Namely, if $-\Delta E_p$ is smaller than $\Delta E_{DP}$, then $\Delta E_k$ is negative, hence the soil mass decreases the speed and comes to a halt if the reserved kinetic energy $E_k$ is all consumed. Thus, provided that the earthquake energy and the energy dissipation mechanism in flowing soil mass are known, it is possible to evaluate the run-out distance in flow-type slides by the energy approach.

In this research, a series of model tests are first carried out to investigate the energy balance in a model slope made from dry sand. The test conditions considered here are much more comprehensive than in the previous research (Kokusho and Ishizawa 2006), including different slope angles and different input frequencies. The test results are then compared with a simple Newmark-type rigid block model to develop an evaluation method for slope deformation based on the energy concept. Finally, the evaluation method is applied to hypothetical slopes.
Shake Table Model Tests

A spring-supported shaking table shown in Fig. 1(a) was utilized to test a model slope made from sand, called Model A, in a rectangular lucite box 80 cm in length, 50 cm in height, and 40 cm in width. The model slope was made from dry clean Toyoura sand (total mass 30 kg) of average particle size $D_{50} = 0.17$ mm and relative density $D_r \approx 40\%$. The sand was air pluviated from a funnel with a constant drop height. The slope angle was parametrically changed at 29, 20, and 10°, considering the angle of repose of the same model slope (35.4°) as explained later. The table was initially pulled to several different horizontal displacements and then released to generate decay-free vibration. The frequency of the vibration was changed in four steps, from 2.7 to 2.5, 2.2, and 2.0 Hz by attaching 1–3 additional steel plates of the same mass (25 kg each) to the table.

Dissipated energy, which can be calculated from displacement amplitudes in the decay vibration, depends not only on the energy dissipation due to slope deformation but also on other energy-loss mechanisms such as radiation into the shake table foundation, friction in the springs, etc. In order to exclusively evaluate the dissipated energy due to slope deformation in Model A, a dummy model, called Model B, consisting of a pile of rigid concrete columns, was tested in the same lucite box in the same way [see Fig. 1(b)]. The total weight and the center of gravity were adjusted to be almost identical in the two models to facilitate a reliable comparison between the two models as will be explained later. The concrete columns were fixed to the box by clamps not to allow energy dissipation due to their relative movements.

The decay in amplitudes, measured by a LVD'T displacement gauge in both Models A and B, is exemplified in Fig. 2. Note that the difference in amplitudes grows larger with the number of cycles, although the initial table displacement and the vibration period of the table are almost the same between the two models. It may be reasonable to assume that this difference reflects the greater energy dissipated in Model A (the model slope) due to its deformations, since almost negligible energy dissipation is expected in the rigid concrete columns in Model B. The energy loss per cycle $\Delta W$ can be calculated as

$$\Delta W = 4\pi WD$$

in which $D$ = damping ratio quantified from decayed vibration exemplified in Fig. 2. The value $W$ in Eq. (3), representing the stored energy in the corresponding cycle, can be evaluated from the spring constant $k$ and the displacement amplitude $u$ of the shaking table as $W = ku^2/2$. The earthquake energy increment dissipated in the model slope $\Delta E_{EQ}$ in Eq. (2) is evaluated from the energy loss per cycle in Model A, $\Delta W_A$, and that in Model B, $\Delta W_B$ as

$$\Delta E_{EQ} = \Delta W_A - \Delta W_B$$

because the energy loss in the two models can be assumed identical except that dissipated inside the sand slope. The total energy $E_{EQ}$ calculated as a sum of $\Delta E_{EQ}$ in each cycle represents the amount of earthquake energy involved in producing the residual displacement in the model slope.

The total input energy supplied to the shaking table $E_{IP}$ can be calculated from the initial pull displacement $u_0$ and a spring constant of the table $k$ as

$$E_{IP} = \frac{1}{2} ku_0^2$$

Fig. 3 shows the values of $E_{EQ}$ actually measured by the aforementioned method plotted versus the total input energy $E_{IP}$ of the shaking table calculated by Eq. (5) from a number of tests with different input energies and different frequencies. Obviously, $E_{EQ}$ is almost proportional to $E_{IP}$ for each input frequency, indicating that a ratio of the earthquake energy $E_{EQ}$ contributing to slope failure to the total input energy $E_{IP}$ may be assumed almost constant no matter how large the input seismic energy is.

Deformation of the model slope was observed by two video cameras, one from the side and the other from above. Vertical markers made from colored sand were placed at the side of the model [Fig. 1(b)]. On the slope face, dry noodle sticks of 5 cm
length were set up in line. The interval of these markers was 10 cm in the down-slope direction. The deformation was measured in each cycle of the input vibration to obtain incremental residual displacement. The slope deformation was also measured before and after the end of tests by a laser beam displacement sensor and compared with the video data to check their reliability. Fig. 4 exemplifies the slope deformation in one of the test cases. The soil mass of 2–3 cm thickness near the surface deforms essentially in shear mode above the dotted line connecting the solid circles as shown in Fig. 4(a). Also note that the deformation across the model thickness is quite uniform as shown in Fig. 4(b), indicating that the test is almost two-dimensional.

In order to correlate the energies with the residual displacement of the slope, horizontal residual displacement of the slope was calculated from the video images. From the engineering point of view, there may be various definitions of the residual slope displacement of the deformable soil mass: average displacement of all deformed soil mass (δm), average displacement of slope surface (δs), or average displacement of slope toe (δt). Among the three displacements, how to evaluate δm needs some explanations, which is available in the literature (Kokusho and Ishizawa 2006). Each of the definitions may have advantages over the others depending on how the slope displacement is incorporated into hazard assessment or engineering design. In Fig. 5, the three displacements obtained in the same test results are compared to one another, indicating that very consistent proportionalities hold among them. In the discussions hereafter, the average displacement of slope surface (δs) will be dealt with, although the other displacements could be used instead because very stable interrelationships can be recognized between them as shown in Fig. 5.

In Fig. 6, the incremental displacement Δδn is plotted versus number of cycles N. The incremental earthquake energy ΔEeq and the incremental potential energy −ΔδEp are also plotted against N in the same chart. The value −ΔδEp is calculated from the change in the slope surface geometry as.

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**Fig. 3.** Earthquake energy contributing to slope failure (Eeq) versus total input energy (Eip) relationships for different frequencies

**Fig. 4.** Typical failure mode of sand slope by shaking table test (slope angle=29°)

**Fig. 5.** Interrelationships among differently defined slope displacements: δm, δs, and δt

**Fig. 6.** Incremental displacement Δδn and incremental energies ΔEeq and −ΔδEp plotted versus the number of cycles N
Fig. 7. Earthquake energy $E_{EQ}$ plotted versus residual slope displacement $\delta_r$, for different input frequencies

$$\Delta \delta_{SP} = \Delta \left( pgB \int zdxdz \right)$$

(6)

where $\rho$ = soil density (assumed constant); $g$ = acceleration of gravity; and $B$ = thickness of the two-dimensional model. Coordinates $x$ and $z$ are in horizontal and vertical directions of the slope as indicated in Fig. 1(a) and the integration in terms of $x$ and $z$ is carried out over the cross-sectional area of the slope. It is confirmed that measurable slope deformations occur only until the fifth cycle in this particular case as demonstrated by the residual displacement $\Delta \delta_r$ in Fig. 6, which is almost consistent with the variation in the energy $\Delta E_{EQ}$ or $-\Delta \delta_{SP}$. It indicates that the energy dissipated inside the model slope is almost ignorable unless measurable residual deformation takes place. The incremental energies, $\Delta E_{EQ}$ and $-\Delta \delta_{SP}$, calculated in each cycle are summed up to evaluate the corresponding total energies, $\Delta E_{EQ}$ and $-\Delta \delta_{SP}$. Then, the dissipated energy $E_{DP}$ can be readily evaluated from Eq. (1) in which $E_i = 0$ if the energy balance after the end of slope failure is concerned.

The total residual displacement $\delta_r$ is also calculated by summing up all incremental displacements $\Delta \delta_r$. In Fig. 7 the residual displacements $\delta_r$ are plotted versus the vibration energy $E_{EQ}$ contributing to slope deformations for three different slope angles of 29°, 20°, and 10° under four different input frequencies. It is remarkable that, for each slope angle, all plots can be approximated as a single curve despite the difference in the input frequency, indicating that the energy can serve as a unique determinant for slope displacement even under different shaking frequencies. Obviously, the gentler the slope is, the greater is the energy $E_{EQ}$ to attain the same residual displacement $\delta_r$. Also noted in Fig. 7 is that there seems to exist a threshold energy, corresponding to each slope angle, below which no residual displacement occurs, indicating that the energy determines not only residual displacements but also the initiation of slope failure.

In order to emphasize the uniqueness of the displacement versus energy relationship, the same residual displacement data of the 29° slope are plotted versus maximum accelerations $A_{max}$ or maximum velocities $V_{max}$ in place of the energy in Figs. 8(a) and (b), respectively. Here, $A_{max}$ represents the acceleration in the first cycle of the decay-free vibration and $V_{max}$ is calculated from $A_{max}$ using the frequency $f_i$ as $V_{max} = A_{max}/(2\pi f)$. Obviously, the same acceleration results in different residual displacements under different input frequencies despite some data scatters, indicating that acceleration cannot be a unique determinant for slope failure not only for the residual slope displacement but even for the initiation of failure. In contrast, Fig. 8(b) implies that velocity can be a better indicator for the slope displacement than acceleration as often pointed out in earthquake engineering. However, some differences can still be recognized for different input frequencies, probably reflecting the fact that not a single velocity value but its accumulation is important in generating residual displacements, which may explain why the energy can be so effective to uniquely determine the residual slope displacement.

Fig. 8. Maximum (a) acceleration or (b) velocity plotted versus residual slope displacement for different input frequencies

Test Data Interpretation by Rigid Block Model

The Newmark method based on a rigid block model or its modifications, a commonly accepted practice in geotechnical earthquake engineering to estimate seismically induced residual displacement of earth structures, has recently been examined from the viewpoint of energy by Kokusho et al. (2004a) and Kokusho and Ishizawa (2006). The application of the energy approach to the rigid block shown in Fig. 9 gives the potential energy change $-\Delta E_r$ and the dissipated energy due to the block slippage $E_{DP}$ to be correlated with horizontal residual displacement $\delta_r$ as
Horizontal Inertia force

Horizontal displacement: \( \delta_x \)

Friction coefficient: \( \mu \)

Horizontal area: \( A \)

\[ \mu = \tan \theta \] (slope inclination; \( \theta \) = friction angle)

\[ \beta = \tan \theta \] (friction angle)

\[ \beta = \tan \theta \]

\[ \mu = \tan \phi \] (friction angle)

\[ E_{DP} = \frac{\mu(1 + \beta^2)}{1 + \mu\beta} M g \delta_x \] (8)

\[ -\delta E_p = \beta M g \delta_x \] (7)

where \( M \) = mass of sliding soil block; \( \beta = \tan \theta \) (slope angle) = slope inclination; and \( \mu = \tan \phi \) (friction angle) = friction coefficient.

Then, starting from Eq. (1) and using \( E_s = 0 \) if compared before and after slope failure, the earthquake energy is correlated with \( \delta_x \), as

\[ E_{EQ} = \delta E_p + E_{DP} = \frac{\mu - \beta}{1 + \mu\beta} M g \delta_x \] (9)

The ratios of \( -\delta E_p \) and \( E_{DP} \) to \( E_{EQ} \) are

\[ \frac{-\delta E_p}{E_{EQ}} = \frac{\beta(1 + \mu\beta)}{(\mu - \beta)} \] (10a)

\[ \frac{E_{DP}}{E_{EQ}} = \frac{\mu(1 + \beta^2)}{(\mu - \beta)} \] (10b)

Note that the contribution of the earthquake energy in comparison to the dissipated energy or the potential energy depends only on \( \beta \) and \( \mu \). Also note that the contribution of \( E_{EQ} \) becomes larger than that of \(-\delta E_p\) with decreasing slope inclination \( \beta \) (Kokusho et al. 2004a).

In these relationships, dynamic changes of seismic inertia force affect not only the driving force of the sliding block but also the shear resistance along the slip surface. If the slip plane is saturated, however, it should be assumed that the seismic inertia force is all carried by temporary pore-water pressure and does not change the effective stress normal to the plane, and hence the shear resistance. In this case, it is easy to understand that the dissipated energy \( E_{DP} \) can be expressed as the shear resistance along the slip plane, \( \mu \sigma_{so} (1 + \beta^2)^{1/2} A \), multiplied by the displacement along the slip plane, \( (1 + \beta^2)^{1/2} \delta_x \), where \( \sigma_{so} \) = effective stress normal to the plane and \( A \) = horizontal area of the sliding soil mass. Consequently, for the saturated slip plane, Eqs. (7) and (8) are replaced by Eqs. (11) and (12), by also substituting \( M g = \sigma_{so} A (1 + \beta^2) \) in Eq. (7), where \( \sigma_{so} \) = total stress normal to the slip plane

\[ -\delta E_p = \beta \sigma_{so} \delta_x (1 + \beta^2) \] (11)

\[ E_{DP} = \mu \sigma_{so} \delta_x (1 + \beta^2) \] (12)

Then, Eq. (13) is obtained in place of Eq. (9)

\[ E_{EQ} = (1 + \beta^2)(\mu \sigma_{so} - \beta \sigma_{so}) A \delta_x \] (13)

The energy ratios, Eqs. (10a) and (10b), are replaced by Eqs. (14a) and (14b)

\[ \frac{-\delta E_p}{E_{EQ}} = \frac{\beta \sigma_{so}}{(\mu \sigma_{so} - \beta \sigma_{so})} \] (14a)

\[ \frac{E_{DP}}{E_{EQ}} = \frac{\mu \sigma_{so}}{(\mu \sigma_{so} - \beta \sigma_{so})} \] (14b)

It is needless to say that the rigid block model cannot exactly reproduce the failure of the deformable sand slope. One of the most significant differences is that, in actual slope failures, soil mass may not slide as a rigid body along a fixed slip plane but deforms continuously with or without movable slip planes as can be observed in Fig. 4(a). It is interesting, therefore, to examine how much the failure mechanism in the sand slope can be captured by the rigid block model.

In Fig. 10, the values of \(-\delta E_p\) and \(E_{DP}\) are plotted versus earthquake energy \(E_{EQ}\) for a number of test data for the 29° slope with different initial table displacements under different input frequencies. In the light of the energy considerations on the rigid block model, the ratios \(-\delta E_p/E_{EQ}\) and \(E_{DP}/E_{EQ}\) can be calculated theoretically by Eqs. (10a) and (10b), in which the friction coefficient is needed. In order to evaluate the friction coefficient \( \mu \) of the model slope, static tests were carried out in which the slope was gradually inclined statically until the slope failed. The tests carried out six times for the same model slope gave the angle of repose 34.5°-36.2° (average 35.4°). A typical failure mode in the static test for the angle of repose is shown in Fig. 11. Substituting the friction coefficient \( \mu = \tan(35.4°) = 0.71 \) and \( \beta = \tan(29°) = 0.55 \) into Eqs. (10a) and (10b), \(-\delta E_p/E_{EQ} = 4.92\) and \(E_{DP}/E_{EQ} = 5.92\) are obtained, which are drawn in Fig. 10 by two dashed lines. Obviously, there is a wide gap between the theoretical lines using the angle of repose and the experimental results. However, if \( \mu = 0.86 \) is used instead of \( \mu = 0.71 \), the theory can predict the test results almost perfectly as shown with the solid lines in Fig. 10.

In Fig. 12, the values of \(-\delta E_p\) are plotted versus earthquake energy \(E_{EQ}\) for a number of test data for the three slope angles,
Fig. 11. Typical failure mode of statically inclined sand slope

Fig. 12. Earthquake energy $E_{EQ}$ plotted versus potential energy for different slope angles and frequencies compared with rigid block theory

Fig. 13. Relationships between residual displacement $\delta_r$ and normalized earthquake energy $E_{EQ}/Mg$ for different slope angles compared with the rigid block theory

In Fig. 13, the residual displacements $\delta_{rs}$, which are considered here to be equivalent to $\delta_r$, in the rigid block model, obtained by a number of tests for different slope angles and different input frequencies, are plotted versus normalized earthquake energies $E_{EQ}/Mg$. The weight of the displaced soil mass $Mg$ was evaluated from Eq. (7) using the measured potential energy $-\delta E_p$ and the measured displacement $\delta_m$ to comply with the rigid block theory. It further implies that, whichever displacement $\delta_{rs}$, $\delta_m$, or $\delta_r$, introduced previously is taken, the value of $Mg$ can be determined automatically to be consistent with the rigid block theory. The dashed lines corresponding to Eq. (15) by the rigid block model for $\mu=0.71$ considerably overestimates the observed residual displacements under the same normalized energy. It is remarkable, however, that if $\mu=0.86$ is chosen again, Eq. (15) can predict the residual slope displacement almost perfectly for all slope angles and all input frequencies. This indicates that if an appropriate friction coefficient is known in advance, the simple rigid block model, which apparently involves the failure mechanism quite differently from reality, can successfully simulate realistic sand slopes.

### Energy-Based Slope Failure Evaluation Method

Based on the model test results and their interpretation by the rigid block theory, an energy-based design method for earthquake-induced residual displacement can be proposed as shown in Fig. 14. First, a sloping ground is idealized by an equivalent horizontal two-layer system consisting of an upper layer including the slope and a base layer as shown in Fig. 15. The input energy $E_{IP}$ transmitting upward is designated at the base layer. In general, assuming one-dimensional (1D) wave propagation of the SH wave, the wave energy $E$ passing through a unit horizontal area $A$ can be formulated (Kokusho et al. 2004b) as
Evaluate incident energy at a base layer $E_{IP}$

Radiation damping energy $E_d$

Energy used for sloping ground $E_{EQ} = E_{IP} - E_d$

Dissipated energy for soil damping, liquefaction, etc. $E_{EQ}$

Dissipated energy for slope failure $E_{EQ} - E_{EQ'}$

$E_{EQ} - E_{EQ'} > E_s$

No

Yes

Friction coefficient $\mu$

Slope inclination $\beta$

Residual slope displacement

\[ \delta_s = \frac{(1 + \mu \beta)(E_{EQ} - E_{EQ'})}{Mg} \]

\[ \delta_s = \frac{(E_{EQ} - E_{EQ'})}{Mg(\mu \beta (\alpha \eta + \beta \sigma))} \]

Fig. 14. Flow chart for evaluation of slope displacement by energy approach

\[ E/A = \rho V_S \int \dot{u}^2 dt \]  
(16)

where $u$ = particle velocity of a wave propagating in one direction in a soil layer; and $\rho V_S$ = impedance of the layer ($\rho$ = soil density and $V_S$ = S-wave velocity). By introducing the energy radiating downward in the base layer, $E_d$, the earthquake energy, $E_{EQ}$, which is dissipated in the upper sloping ground, can be obtained as

\[ E_{EQ} = E_{IP} - E_d \]  
(17)

In the present test series, the ratio $E_{EQ}/E_{IP}$ was found almost constant as shown in Fig. 3 despite the difference in the magnitude of $E_{IP}$. In order to have some insights into this experimental finding, let us consider an energy flow in the two-layer model of Fig. 15. S-wave velocities are $V_{S1}$ and $V_{S2}$, and soil densities are $\rho_1$ and $\rho_2$ in the upper and base layers, respectively. If a harmonic wave is given in the base layer and propagated in the vertical direction ($z$ direction), a stationary response of the system can be written as

\[ u_1 = A_1 e^{i(\omega t - k_1 z)} + B_1 e^{i(-\omega t + k_1 z)} \]

\[ u_2 = A_2 e^{i(\omega t - k_2 z)} + B_2 e^{i(-\omega t + k_2 z)} \]  
(18)

where $u_1$ and $u_2$ = dynamic displacements; $k_1$ and $k_2$ = wave numbers defined by $k_1 = \omega / V_{S1}$ and $k_2 = \omega / V_{S2}$ in the upper and base layers, respectively; $i = \sqrt{-1}$; $t$ = time; and $\omega$ = angular frequency.

Constants $A_1$ and $B_1$ = wave amplitudes for the upward and downward waves in the upper layer; and $A_2$ and $B_2$ = amplitudes of the upward incident wave and the downward radiating wave in the base layer, respectively.

In an extreme case in which the energy dissipation in the upper layer becomes large enough, it seems possible to assume that the amplitude of the wave propagating downward in the upper layer is almost zero; $B_1 = 0$. Then, from the continuity of horizontal deformation and shear stress at the boundary, the constants $A_1$ and $A_2$ can be correlated (Kokusho et al. 2004c) as

\[ A_1 / A_2 = 2(1 + \alpha) \]  
(19)

where $\alpha$ = impedance ratio defined by $\alpha = \rho_1 V_{S1} / \rho_2 V_{S2}$. Based on Eq. (16), it is easy to understand that the energy ratio is expressed as the square of the amplitude ratio in Eq. (19) times the impedance ratio $\alpha$. Hence, the corresponding energy ratio is written as

\[ E_{EQ}/E_{IP} = 4\alpha/(1 + \alpha)^2 \]  
(20)

Thus, by assuming that no downward wave exists because all the energy transmitting into the upper layer is completely dissipated inside the upper layer due to slope failure, energy ratio $E_{EQ}/E_{IP}$ can be quantified by Eq. (20).

This proportionality between the two energies seems to be consistent with the aforementioned experimental observation in Fig. 3 that $E_{EQ}/E_{IP}$ was actually almost constant. According to Eq. (20), the value of $E_{EQ}/E_{IP} = 0.19 - 0.25$ in Fig. 3 for the model slopes corresponds to a quite low impedance ratio; $\alpha = 0.053 - 0.072$, presumably due to the sharp contrast of impedance between the model sand slope and the shaking table consisting of steel plates. In the test, the slope failure occurred from the first cycle of the shaking motion, justifying the assumption that most of the energy was consumed by the slope failure from the first and no significant energy return to the table occurred. Eq. (20) may well be expected to hold also in situ to some degree, although it should be considered to give an upper limit. In actual

\[ \rho_1 V_{S1} \]

\[ A_1 \]

\[ B_1 \]

\[ \rho_2 V_{S2} \]

\[ A_2 \]

\[ B_2 \]

\[ E_{IP} \]

\[ E_d \]

Fig. 15. Idealization of sloping ground by equivalent horizontal two-layer system and wave propagations in it
problems, the ground may consist of multiple layers in which a design input motion is given at a base layer. The energy ratio $E_{EQ}/E_{IP}$ may then be evaluated from a multi-reflectance analysis of the one-dimensional model in which the damping ratio of the top layer, where the slope failure is expected to occur, is assumed extremely large.

Earthquake energy $E_{EQ}$ to be used in actual design is composed of two orthogonal seismic motions and cannot be separated in advance into components parallel or perpendicular to sloping directions. Among the two, the energy by the perpendicular motion will be involved in deteriorating soil properties but not directly involved in the energy balance of the slope in Eq. (1). However, for engineering purposes, it may be practical to use the total energy for conservative evaluations.

Out of energy $E_{EQ}$ supplied in the upper layer, a part of it seems to be dissipated by cyclic straining of soil or internal soil damping, although its contribution was almost negligible in the model test presumably because the model sand slope was highly confined at the vertical boundary through the slope crest and difficult to deform in the shear mode. If this cyclically dissipating energy is denoted by $E_{EQ}$, then the energy used exclusively for slope failure becomes $(E_{EQ} - E_{EQ})$.

The present test results in Fig. 7 demonstrated that not only the slope displacement but also the initiation of slope failure are controlled by threshold earthquake energy rather than by acceleration. Consequently, if the value $(E_{EQ} - E_{EQ})$ goes beyond some threshold, $E_{EQ} - E_{EQ} > E_{cr}$, then the slope failure is to be judged to occur. In the current state of the art, however, there exists no reasonable method to determine the threshold energy, $E_{cr}$, and further research is needed in this direction. In the mean time, a conventional judgment by acceleration on the initiation of slope failure may be used.

If a slope is judged to slide, residual horizontal displacement is calculated based on Eq. (9) as

$$\delta_i = \frac{(1 + \mu \beta)(E_{EQ} - E_{EQ})}{(\mu - \beta)M_g}$$

(21)

This equation is applicable to unsaturated slopes where seismic inertia affects not only the driving force but also the shear resistance along the slip plane. If a slip plane is saturated, then the following equation based on Eq. (13) should be used:

$$\delta_i = \frac{(E_{EQ} - E_{EQ})/A}{(1 + \beta^2)(\mu\gamma_{sat} - \beta\gamma_{sat})}$$

(22)

In general, the thickness or the mass of sliding soil necessary for Eq. (21) or (22) may be determined by conventional slip surface analyses to look for a potential slip surface with a minimum factor of safety. In some cases, the potential slip surface may be reasonably assumed to coincide with a bedding plane or a seam of weak soils considering case histories during previous earthquakes such as the 2004 Niigataken Chuetsu earthquake (e.g., Kokusho and Ishizawa 2005a).

If the displacement or the run-out distance of failed soil mass gets larger, the slip plane may vary its slope angle or friction coefficient in its down-slope direction. In such cases, it is necessary to divide the slip plane into sections and examine the energies in each section according to the original energy balance. Namely, if the kinetic energy calculated by Eq. (1) at the end of the first section is positive, then the soil mass overruns to the second section. In that case, the residual kinetic energy at the end of the first section should be transferred to the next section to be included in the energy balance there, the detail of which will be discussed later.

**Case Study on Hypothetical Slopes**

In order to demonstrate a basic applicability of the energy approach and obtain some insights on run-out distance expected during extremely strong earthquakes, a case study on hypothetical slopes is carried out here following the flow chart in Fig. 14. First, a simple slope shown in Fig. 16(a) is considered. Upper ground consists of a sliding soil block and a simple slope of inclination $\beta$ overlying a base layer. The impedance ratio between the upper and base layers is assumed as $\alpha = \rho_1 V_{S1}/\rho_2 V_{S2} = 0.25$. The soil block is considered to slide along a straight slip plane, like a large-scale dip slip that occurred during the 2004 Niigataken Chuetsu earthquake (Kokusho and Ishizawa 2005a), due to input earthquake energy $E_{IP}/A$ defined at the base layer. The horizontal area of the block is $A$ (length $L$ and unit widths; $A = L \times 1$) and its total weight is $M_g = \rho g HL$, where $\rho$ = soil density and $H$ = average thickness of the block. The soil block is assumed totally unsaturated in one case or locally saturated only along the slip plane in another case.

As the first step of the slope displacement evaluation, the earthquake energy has to be determined. In order to estimate the incident wave energy for strong earthquakes, let us review the energies for the 1995 Kobe earthquake ($M_L=7.2$ in Japanese earthquake magnitude almost equivalent to Richter's magnitude) evaluated by Kokusho and Motoyama (2002). The input energy per unit area $E_{IP}/A$ at a base layer during that earthquake was evaluated at four vertical array sites (abbreviated as PI, SGK, TKS, and KNK) based on an assumption that the major energy is transmitted by the vertically propagating SH wave. In Fig. 17, the
total energies calculated from two orthogonal horizontal motions are plotted versus the focal distances. Among them, the energy at a base layer of 83.4 m depth at the Port Island site (PI), only a few kilometers away from the causative fault, was the largest ($E_{\text{PI}}/A = 305 \text{ kJ/m}^2$).

The solid line in the same chart indicates the wave energy per unit area theoretically calculated from the earthquake magnitude $M_j$ and the focal distance $R$. Here, the spherical energy radiation of body waves from the hypocenter was assumed by the next equation

$$E_{\text{PI}}/A = E_0/(4\pi R^2) \tag{23}$$

The total wave energy $E_0$ (unit: erg = $10^{-10}$ kJ) in Eq. (23) released at a point source was determined using an empirical equation by Gutenberg (1955)

$$\log E_0 = 1.5M + 11.8 \tag{24}$$

in which $M_0$ = Richter’s magnitude, although $M_j$ is actually used in the computation. The values of $E_{\text{PI}}/A$ calculated from earthquake records are quite variable from site to site, presumably due to fault rupture mechanisms such as directivity effects. Namely, PI and SGK with larger energies are along the fault or its extension, while TKS and KNK are located far off the fault extension line [see Kokusho and Motoyama (2002)]. Nevertheless, it is remarkable that the points with larger energies are located quite near to the theoretical line. Despite great uncertainties involved in the energy calculations due to simplifications on the mechanisms of earthquake fault or wave propagations, it may well be assumed that the maximum values of $E_{\text{PI}}/A$ at a base layer can be roughly approximated by Eq. (23) together with Eq. (24).

After this preparation, let us start the hypothetical case study. Suppose, for instance, a crustal near-field earthquake of $M_j=7.4$ and $R=15$ km occurred near the site. This value of magnitude corresponds to the 1847 Zenko-ji earthquake, which represents one of the largest near-field events historically that occurred in the mountainous area of Japan, triggering countless slope failures. Then, the energy per unit area calculated from Eqs. (23) and (24) is $E_{\text{PI}}/A = 2,810 \text{ kJ/m}^2$, which is much larger than $E_{\text{PI}}/A = 305 \text{ kJ/m}^2$ in PI during the Kobe earthquake and may be nearly an upper extreme to be assumed in engineering design. If the impedance ratio is given as $\alpha = p_1 V_s_1 / p_2 V_s_2 = 0.25$, then $E_{\text{IEF}}/E_{\text{II}} = 0.64$ from Eq. (20) and the maximum earthquake energy per unit area directed to slope deformation or slope failure will be $E_{\text{IEF}}/A = 1,800 \text{ kJ/m}^2$.

Next, energy $E_{\text{IEF}}$ dissipated in the upper layer due to cyclic straining should be considered. It is assumed here for simplicity that the soil block in Fig. 16(a) exclusively develops higher seismic strain leading measurable internal energy dissipation while the energy dissipation in the underlying sloping ground of competent soil can be neglected. There exist several previous researches on dissipated energy in soil samples by means of undrained cyclic loading tests. For example, the dissipated energy per unit volume, $E_{\text{cu}}$, normalized by effective confining stress, $\sigma'_{\text{c}}$, to liquefy saturated sands was measured as $E_{\text{cu}}/\sigma'_{\text{c}} = 0.012-0.032 \text{ kJ/m}^2/\text{kPa}$ in undrained strain-controlled cyclic shear tests on medium dense sand of relative density of $D_r=50-70$% (Fiqueroa et al. 1994). In stress-controlled undrained triaxial tests, the normalized dissipated energy was measured as $E_{c}'_{\text{cu}}/\sigma'_{\text{c}} = 0.01-0.03 \text{ kJ/m}^2/\text{kPa}$ to develop 5% double-amplitude axial strain or $E_{c}'_{\text{cu}}/\sigma'_{\text{c}} = 0.02-0.06 \text{ kJ/m}^2/\text{kPa}$ to develop 10% double-amplitude axial strain on reconstituted gravelly soils with fines content of 0-30% (Hara et al. 2001). On intact sandy soils recovered from an actual slope that failed during the 2003 Sanriku-Minami earthquake ($M_j=7.0$) in northern Japan, the normalized dissipated energy to develop 5% double-amplitude axial strain was quantified as $E_{\text{cu}}'/A = 0.042 \text{ kJ/m}^2/\text{kPa}$ (Kokusho and Ishizawa 2005b). Based on the previous research, if $E_{\text{cu}}'/\sigma'_{\text{c}} = 0.05 \text{ kJ/m}^2/\text{kPa}$ is roughly assumed for the sliding soil block (average height $H=20$ m, density $\rho=2.0 \text{ t/m}^3$, and average effective confining stress $\sigma'_{\text{c}}=200 \text{ kPa}$), the value of dissipated energy per unit area $E_{\text{cu}}'/A = (E_{\text{cu}}'/\sigma'_{\text{c}}) \times \sigma'_{\text{c}} \times H = 0.05 \text{ kJ/m}^2/\text{kPa} \times 200 \text{ kPa} \times 20 \text{ m} = 200 \text{ kJ/m}^2$ is obtained, resulting in the earthquake energy exclusively for slope sliding, $(E_{\text{IEF}}-E_{\text{IEF}})/A = 1,600 \text{ kJ/m}^2$.

The residual displacement in Eq. (21) or (22) can be expressed in a normalized form as

$$\delta/L = \frac{1 + \mu \beta (E_{\text{IEF}}-E_{\text{IEF}})/A}{\mu - \beta} \rho g H L \tag{25}$$

for the unsaturated slip plane, in which the weight of soil mass is $M_g = \rho g H L$, or
Table 1. Parameters for Slope Failure Evaluations of Hypothetical Slopes

<table>
<thead>
<tr>
<th>Type of slope</th>
<th>Incident energy $E_{ip}$ (kJ/m$^2$)</th>
<th>Impedance ratio $\alpha$</th>
<th>Energy for slope failure $(E_{EQ} - E_{EQ}^*)/A$ (kJ/m$^2$)</th>
<th>Soil block length $L$ (m)</th>
<th>Soil block height $H$ (m)</th>
<th>Soil block density $P$ (t/m$^3$)</th>
<th>Slope angle $\theta$ (degrees)</th>
<th>Friction angle $\phi$ (degrees)</th>
<th>Residual displacement $\delta_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>2,800</td>
<td>0.25</td>
<td>1,600</td>
<td>100</td>
<td>20</td>
<td>2.0</td>
<td>15</td>
<td>?</td>
<td>$L/100$, $L/10$, $L$, 10 $L$</td>
</tr>
<tr>
<td>Complex</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\theta_1=30$</td>
<td>$\phi=25$</td>
<td>?</td>
</tr>
</tbody>
</table>

$$\delta_r/L = \frac{1}{\mu - \beta} \left( \frac{E_{EQ} - E_{EQ}^*}{A} \right) \rho g H L$$  \hspace{1cm} (26)

for the saturated slip plane, where $\sigma_{so}^* = \sigma_{so} = \rho g H/(1+\beta^2)$, assuming that only the slip plane and nearby is locally saturated. In preparation to parametrically evaluate the residual displacement of the hypothetical slope, the value of $(1+\mu \beta)/(\mu - \beta)$ or $1/(\mu - \beta)$ in Eqs. (25) and (26) is calculated for the slope angles $\theta=15^\circ$ ($\beta=0.268$), $\theta=30^\circ$ ($\beta=0.577$), $\theta=45^\circ$ ($\beta=1.000$), and $\theta=60^\circ$ ($\beta=1.732$), and shown in Fig. 18 against the normalized friction coefficient $\mu/\beta$. It is seen for the widely varying slope angles that the values drastically increase as $\mu/\beta$ approaches unity. This indicates that the friction coefficient plays a really important role in realizing large displacement flow failures.

In Table 1, pertinent parameters used in the case study on the simple slope of Fig. 16(a) are listed. In order to differentiate the degree of damage, the residual displacement $\delta_r$ is set stepwise from $L/100$ to $10L$, and the friction angle causing that displacement is calculated parametrically for the four steps of slope angles under the condition of $\rho=2.0$ t/m$^3$, $H=20$ m, and $L=100$ m.

Table 2 shows the results of the parametric study. It indicates that, under this extreme seismic energy, it is impossible for the unsaturated slip plane to restrict the displacement within 1% of the soil block length ($\delta_r=1.0$ m), no matter how large $\phi$ or $\mu$ may be. In contrast, 10% displacement ($\delta_r=10$ m) may occur quite probably because the values of $\phi$ seem realistic in comparison with corresponding values of $\theta$ for all the slope angles. However, if 100% displacement ($\delta_r=100$ m) is considered, it is necessary for the $\phi$ value being close to $\theta$ within only $1-2^\circ$ as shown in Table 2. In order to obtain 1000% displacement ($\delta_r=1000$ m), the $\phi$ value has to be almost identical to $\theta$. This indicates that even if the extremely large seismic input is considered, a flow-type failure with a run-out distance almost equal to the soil block length cannot be realized, unless the friction coefficient approaches the slope inclination already before the earthquake (the slope is in the brink of sliding even without seismic effect) or decreases drastically due to seismic loading or shearing effect.

It has been found actually in case studies using the same energy approach on long run-out distance slope failures during recent earthquakes in Japan that back-calculated friction coefficients are smaller than the inclination of sliding planes. In the first case, during the 2003 Sanriku–Minami earthquake ($M_s=7.0$) in northern Japan, equivalent friction coefficient of sliding soil volume of about $6 \times 10^5$ m$^3$ evaluated from a run-out distance of about $\delta_r=100$ m was $\mu=0.15$ while inclination of the simple slip plane was $\beta=0.16$ (Kokusho and Ishizawa 2005b). In the second case, during the 2004 Niihataken Chuetsu earthquake in central Japan, a huge slide of about $1 \times 10^6$ m$^3$ soil volume with the run-out distance of about $\delta_r=90$ m was back-calculated to have $\mu=0.22$ in contrast to a much higher inclination of the slip plane $\beta=0.36$ (Kokusho and Ishizawa 2005a).

Table 3 shows a ratio between the potential energy and the earthquake energy, $-\delta E_p/E_{EQ}$ for the same cases as listed in Table 1. It is noteworthy that, if $\delta_r/L$ is on the order of 0.01, the earthquake energy $E_{EQ}$ contributes more than the potential energy $-\delta E_p$, particularly in the gentle slope of $\theta=15^\circ$. If $\delta_r/L$ is on the order of 0.1, then $E_{EQ}$ and $-\delta E_p$ are comparable for the $\theta=15^\circ$ slope, although in the steeper slopes the contribution of $E_{EQ}$ decreases compared to $-\delta E_p$. For the displacement $\delta_r/L=1.0$ or larger, the effect of $-\delta E_p$ far exceeds that of $E_{EQ}$ for all the slope

Table 2. Friction Coefficients $\mu = \tan \phi$ Necessary to Attain Normalized Residual Displacements $\delta_r/L$ in Hypothetical Case Study on Simple Slope

<table>
<thead>
<tr>
<th>Slope plane</th>
<th>Inclination</th>
<th>Angle (degrees)</th>
<th>$\delta_r/L=0.01$</th>
<th>Angle (degrees)</th>
<th>$\delta_r/L=0.1$</th>
<th>Angle (degrees)</th>
<th>$\delta_r/L=1$</th>
<th>Angle (degrees)</th>
<th>$\delta_r/L=10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsaturated</td>
<td>$\beta=0.267$</td>
<td>$\phi=15$</td>
<td>$\beta=0.758$</td>
<td>$\phi=37$</td>
<td>$\beta=0.311$</td>
<td>$\phi=17.3$</td>
<td>$\beta=0.271$</td>
<td>$\phi=15.2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta=0.577$</td>
<td>$\phi=30$</td>
<td>$\beta=1.289$</td>
<td>$\phi=52$</td>
<td>$\beta=0.633$</td>
<td>$\phi=32.3$</td>
<td>$\beta=0.582$</td>
<td>$\phi=30.2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta=1.000$</td>
<td>$\phi=45$</td>
<td>$\beta=2.379$</td>
<td>$\phi=67$</td>
<td>$\beta=1.085$</td>
<td>$\phi=47.3$</td>
<td>$\beta=1.008$</td>
<td>$\phi=45.2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta=1.732$</td>
<td>$\phi=60$</td>
<td>$\beta=7.303$</td>
<td>$\phi=82$</td>
<td>$\beta=1.908$</td>
<td>$\phi=62.3$</td>
<td>$\beta=1.748$</td>
<td>$\phi=60.2$</td>
<td></td>
</tr>
<tr>
<td>Saturated</td>
<td>$\beta=0.267$</td>
<td>$\phi=15$</td>
<td>$\beta=4.349$</td>
<td>$\phi=77$</td>
<td>$\beta=0.675$</td>
<td>$\phi=34$</td>
<td>$\beta=0.308$</td>
<td>$\phi=17.1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta=0.577$</td>
<td>$\phi=30$</td>
<td>$\beta=6.593$</td>
<td>$\phi=78$</td>
<td>$\beta=0.985$</td>
<td>$\phi=45$</td>
<td>$\beta=0.618$</td>
<td>$\phi=31.7$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta=1.000$</td>
<td>$\phi=45$</td>
<td>$\beta=5.082$</td>
<td>$\phi=79$</td>
<td>$\beta=1.408$</td>
<td>$\phi=55$</td>
<td>$\beta=1.041$</td>
<td>$\phi=46.2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta=1.732$</td>
<td>$\phi=60$</td>
<td>$\beta=5.814$</td>
<td>$\phi=80$</td>
<td>$\beta=2.140$</td>
<td>$\phi=65$</td>
<td>$\beta=1.773$</td>
<td>$\phi=60.6$</td>
<td></td>
</tr>
</tbody>
</table>

Note: Earthquake magnitude: $M=7.4$; $R=15$ km; $E_{ip}/A=2,810$ kJ/m$^2$; and $(E_{EQ} - E_{EQ}^*)/A=1,600$ kJ/m$^2$.
angles by tens or hundreds of times. Thus, Table 3 clearly indicates that even the largest earthquake energy corresponding to near-field extreme events is actually far smaller than the potential energy of long run-out distance failures for large failed soil mass considered in the hypothetical slope. This further implies that the earthquake energy plays a major role in initiating the slope failure by supplying an additional energy for destabilization and also in reducing shear resistance in the sliding soil mass by cyclic loading effect rather than in contributing directly to the energy needed for long run-out failures.

Next, in order to evaluate long run-out distance in a complex slope by the energy method, let us consider the second hypothetical slope shown in Fig. 16(b) consisting of two sections of unsaturated slope with different inclinations, $\beta$ and $\beta'$ (greater than $\beta$), and the horizontal distance from the initial center of the sliding mass to the section boundary is 5L. ($L=100 m$). The pertinent parameters needed for the calculation are listed in the bottom line of Table 1. The soil block of the same condition is supposed to slide along the same slip plane of $\theta=30^\circ$ due to the same input earthquake energy as in the simple slope. If the friction angle is assumed to somehow decrease to be smaller than the slope angle, $\phi=25^\circ < \theta=30^\circ$, the sliding soil mass will accelerate and exceed into the second section with a certain speed. Then, the kinetic energy at the boundary per unit horizontal area is calculated based on Eq. (1) as

$$ E_k/A = (E_{eq} - E_{ip})/A - \delta E_p/A - E_{dp}A $$

(27)

If $\beta=0.577$ ($\theta=30^\circ$) and $\beta'=0.257$ ($\theta=15^\circ$), and the friction coefficient $\mu=0.466$ ($\phi=25^\circ$) is assumed all along the planes, for instance, the energies on the right-hand side of Eq. (27) are $(E_{eq} - E_{ip})/A=1,600 kJ/m^2$ as given before, $-\delta E_p/A=113,200 kJ/m^2$ from Eq. (7) and $E_{dp}A=96,000 kJ/m^2$ from Eq. (8) in the first section. Thus, $E_k/A=18,800 kJ/m^2$ is transferred to the second section. If the earthquake shaking already finishes by that time ($E_{eq}/A=0$), then the energy balance in the second section will hold based on Eqs. (7) and (8) as

$$ E_k/A = E_{dp}A - (-\delta E_p/A) = \frac{\mu - \mu'}{1 + \mu B} \rho g H \delta_x $$

(28)

where $E_k/A=E_k/A=18,800 kJ/m^2$ = kinetic energy per unit area transferred from the first section. The run-out distance in the second section is obtained from Eq. (28) as $\delta_x=257 m$.

Thus, the hypothetical case studies have demonstrated that the energy approach can potentially serve as a powerful tool for predicting residual displacements including long run-out distances in seismically induced slope failures. However, calculations described here are based on several assumptions that still need close scrutiny for application to slopes in the field. Among them, how to determine an appropriate friction coefficient corresponding to actual slopes is most essential. For that goal, model tests and case history studies for actual slope failures under various conditions are needed in order to back-calculate friction coefficients particularly for cases of long run-out distance.

Conclusions

An energy approach for slope failure evaluation has been developed by first conducting a series of innovative shake table model tests of dry sand slopes and then examining the associated energy balance by comparing with a Newmark-type rigid block model. An energy-based evaluation method is then proposed and applied to hypothetical slopes shaken by extremely large earthquake energy to clarify critical conditions for flow-type slope failure to occur. Major findings obtained in this research are as follows:

1. In shake table tests of dry sand slopes with different slope inclinations and different input frequencies, earthquake energy $E_{eq}$ to be directed to slope failure could be successfully measured, quantifying the energy balance involved in the failure of the model slopes.

2. The model tests yielded a unique relationship between energy $E_{eq}$ and residual slope displacement $\delta_s$ for each slope inclination that is independent of input frequency. In contrast, acceleration cannot uniquely determine not only the displacement but also the initiation of the failure.

3. The above-mentioned $E_{eq}$ versus $\delta_s$ relationship shows a clear threshold of $E_{eq}$ below that $\delta_s=0$, which is again independent of input frequency. This implies that not only the residual displacement but also the initiation of slope failure may be determined uniquely by the energy.

4. Comparison of the test results with the energy balance in a Newmark-type rigid block theory indicates that the rigid block model, which apparently possesses a different failure mechanism, can almost perfectly emulate a continuously deforming sand slope provided that an appropriate friction coefficient $\mu$ can be estimated.

5. A flow chart for evaluating residual slope displacement was proposed in which the displacement is evaluated from slope inclinations and friction coefficients by considering incident wave energy given at a base layer and energy dissipation in an upper layer. It was also indicated that maximum dissipated energy due to slope failure may be given in proportion to incident wave energy as a simple function of the impedance ratio between the two layers.

6. A case study on hypothetical simple slopes indicates that, even under an extremely large earthquake, flow-type slope failure of a long run-out distance cannot occur unless friction coefficient approaches slope inclination. Also indicated is that even the strongest earthquake energy is actually far smaller than the potential energy as far as long run-out dis-
tance slope failure of large soil mass is concerned.

7. The finding in Conclusion 6 implies that earthquake energy plays a major role in initiating slope failure by supplying an additional energy for destabilization and also in reducing shear resistance in the sliding soil mass rather than in contributing directly to the energy-balance in the failure of large soil mass. Thus, the energy-based failure initiation mechanism and the reduction mechanism in shear resistance are really important in evaluating potential hazards of seismically induced long run-out distance slope failures.

Acknowledgments

The undergraduate students of the Civil Engineering Department in Chuo University who were involved in this research are gratefully acknowledged for their great help in conducting shaking table tests and processing the data.

Notation

The following symbols are used in this paper:

- $A$ = horizontal area of sliding soil mass;
- $A_{1}, A_{2}$ = amplitudes for upward waves in upper and base layers, respectively;
- $A_{\text{max}}$ = maximum acceleration in the first cycle in decayed free vibration of shake table;
- $B$ = thickness of two-dimensional slope model;
- $B_{1}, B_{2}$ = amplitudes for downward waves in upper and base layers, respectively;
- $D$ = damping ratio;
- $E_{c}$ = threshold energy for slope failure initiation;
- $E_{d}$ = energy radiating downward in base layer;
- $E_{dp}$ = dissipated energy in sliding soil mass;
- $E_{\text{EQ}}$ = earthquake energy or vibration energy contributing to slope failure;
- $E_{\text{EQ}}'$ = dissipated energy by cyclic straining of soil or internal soil damping;
- $E_{ip}$ = input energy supplied to base of slope;
- $E_{k}$ = kinetic energy;
- $E_{k}'$ = kinetic energy in second the slope transferred from the first;
- $E_{0}$ = total wave energy released from point source;
- $E_{iv}$ = dissipated energy per unit volume to liquefy saturated sands;
- $f$ = frequency in decayed free vibration of shake table;
- $g$ = acceleration of gravity;
- $H$ = average thickness of sliding soil block;
- $k_{1}, k_{2}$ = wave numbers in upper and base layers, respectively, defined by $k_{1}=\omega V_{S_{1}}$, $k_{2}=\omega V_{S_{2}}$;
- $L$ = horizontal length of sliding soil block;
- $M$ = mass of sliding soil block;
- $M_{j}$ = Japanese earthquake magnitude almost equivalent to the Richter’s magnitude;
- $M_{R}$ = Richter’s earthquake magnitude;
- $R$ = focal distance;
- $u$ = displacement amplitude of spring-supported shake table;
- $\dot{u}$ = particle velocity of wave propagating in one direction;
- $u_{0}$ = initial displacement amplitude of spring-supported shake table;
- $u_{1}, u_{2}$ = dynamic displacements in upper and base layers, respectively;
- $V_{\text{max}}$ = maximum velocity in the first cycle in decayed free vibration of shake table;
- $V_{S}$ = S-wave velocity;
- $W$ = stored energy per cycle in shake table test;
- $x$ = horizontal coordinate from slope edge;
- $z$ = vertical coordinate from slope bottom;
- $\alpha$ = impedance ratio defined by $\alpha=p_{1}V_{S_{1}}/p_{2}V_{S_{2}}$;
- $\beta$ = slope inclination ($\beta=\tan \theta$);
- $\beta'$ = second slope inclination in complex slope;
- $\delta_{v}$ = residual horizontal slope displacement of rigid block;
- $\delta_{v_{a}}$ = average displacement of all deformed soil mass;
- $\delta_{v_{r}}$ = average displacement of slope surface;
- $\delta_{v_{t}}$ = average displacement of slope toe;
- $-\Delta E_{p}$ = potential energy change by slope failure;
- $\Delta W$ = loss energy per cycle in shake table test;
- $\theta$ = slope angle;
- $\kappa$ = spring constant of spring-supported shake table;
- $\mu$ = friction coefficient ($\mu=\tan \theta$);
- $\rho$ = soil density;
- $\sigma_{e}$ = effective confining stress;
- $\sigma_{n}$ = total stress normal to slip plane;
- $\sigma_{s}$ = effective stress normal to slip plane;
- $\phi$ = friction angle; and
- $\omega$ = angular frequency.

References


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