Seismic Amplification between Base and Surface using Average Vs in Equivalent Surface Layer

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ABSTRACT:

Site amplification essential for seismic zonation between ground surface and a base layer was investigated using surface and base accelerations recorded in a number of KiK-net downhole arrays in Japan during recent destructive earthquakes. Based on soil investigation data available for individual arrays, theoretical amplifications for surface arrays consistent with the vertical array records were calculated. A good and unique correlation was found between the peak amplification thus obtained and S-wave velocity ratio, defined as S-wave velocity in a base layer divided by average S-wave velocity, \(V_s\) for different sites and different earthquakes. The value of \(V_s\) was evaluated from fundamental mode frequency and a thickness of equivalent surface layer in which peak amplification was exerted. The conventional parameter \(V_{30}\) often used in current practice, showed poorer correlation than \(V_s\) with the obtained amplifications. It is suggested that \(V_s\) may be determined from microtremor measurements.

INTRODUCTION

Site amplification is defined between ground surface and bedrock and depends on several factors; the composition of soil layers, S-wave velocities, soil densities, internal damping of the individual layers. Furthermore, strain-dependent nonlinear properties may affect site response in soft soils during strong earthquakes.

Among the influencing factors, a ratio of S-wave velocity between the base layer and sur-
face soil is of utmost importance. Shima (1978) found that the analytically calculated site amplification is almost linearly related to the ratio of S-wave velocity of the surface top layer to that of base layer despite the difference in intermediate soil layers.

In order to evaluate site amplification from a base layer to ground surface, NIED (National Research Institute for Earth Science and Disaster Prevention, Japan) deployed several hundreds of vertical array strong motion recording systems called KiK-net all over Japan, which comprises a pair of 3D accelerometers at ground surface and base layer. The observed records together with associated geotechnical data are easily accessible by international researchers at the web site; http://www.kik.bosai.go.jp/kik/.

In general, there can be two different seismic array systems to measure the site amplification between ground surface and base layer as illustrated in Figs. 1(a) and (b). One is a surface array (a) consisting of a set of surface seismometers on different surface geologies in a relatively small area with a common base layer. Seismic records \(2A_s\) on a surface soil and \(2A_b\) on outcropping base layer allow to directly evaluate site amplification between the two different geologies \(2A_s/2A_b\) if incident seismic wave \(A_b\) is assumed basically the same in that area. The other is a vertical array (b) consisting of surface and down-hole seismometers at the same place. This can evaluate site amplification exactly at the same location, though some modification is necessary to extract the outcrop motion \(2A_b\) from observed base motion \((A_b+B_b)\) which is more or less contaminated by downward wave \(B_b\) from overlying layers.

In the context, Kokusho and Sato (2008) evaluated spectrum amplifications from strong motion records obtained by the KiK-net vertical arrays during 3 strong earthquakes occurred in recent years in Japan. The Fourier spectrum ratio \(2A_s/(A_b+B_b)\) between the two measured motions at surface and base first calculated was used to modify the theoretical transfer function \(2A_s/2A_b\) based on multiple reflection theory of 1-dimensional SH waves by removing the effect of downward wave \(B_b\) on the measured motion at the base layer. The peak amplifications of the modified transfer functions \(2A_s/2A_b\) were then correlated with S-wave velocity ratios between base and surface which was newly proposed in the research. In this paper, the number of earthquakes is increased from 3 to 8 to investigate if their spectrum amplifications are correlated in the similar manner with the S-wave velocity ratios.

EARTHQUAKE RECORDS AND AVERAGE S-WAVE VELOCITY

In addition to three earthquakes (EQ1: the 2003 Tokachi-Oki earthquake \((M_f=8.0)\), EQ2:
the 2004 Niigataken-Chuetsu earthquake (Mj=6.8), EQ3: the 2005 Fukuokaken-Seihou-Oki earthquake (Mj=7.0), five new earthquakes are used here; EQ4: the 2000 Tottoriken-Seibu earthquake (Mj=7.3), EQ5: the 2001 Geiyo earthquake (Mj=6.4), EQ6: the 2007 Niigataken-Chuetsu-Oki earthquake (Mj=6.8), EQ7: the 2007 Noto-Hanto earthquake (Mj=6.9) and EQ8: the 2008 Iwate-Miyagi-Nairiku earthquake (Mj=7.2). Here, Mj is the earthquake magnitude on the Japanese Meteorological Agency scale and nearly equivalent to the Richter Magnitude. Strong motion records at 20 sites with peak ground acceleration (PGA) higher than 200 cm/s^2 in EQ1, at 15 sites with PGA >100 cm/s^2 in EQ2, at 11 sites with PGA >100 cm/s^2 in EQ3, at 20 sites with PGA >100 cm/s^2 in EQ4, at 17 sites with PGA >100 cm/s^2 in EQ5, at 10 sites with PGA >100 cm/s^2 in EQ6, at 5 sites with PGA >100 cm/s^2 in EQ7, and at 17 sites with PGA >150 cm/s^2 in EQ8 are used in this research. As a whole, the PGA-values span from 100 to 2500 cm/s^2, while maximum accelerations at the base are less than 200 cm/s^2, showing approximately 2-10 times amplification of the peak horizontal accelerations.

The depths of the down-hole seismometers in the vertical arrays used here vary from 100 m to 330 m except for 1 site (800 m). The base layers where down-hole seismometers are installed has S-wave velocities Vs which normally stable in the depth and do not drastically change between neighboring layers. However, the Vs values diverge from Vs = 400 m/s to 3000 m/s among different sites in the areas of the eight earthquakes. In a good contrast, the surface velocity Vs is around 200 m/s on average for most of the recording sites used here.

Fourier spectrum ratio was computed between ground surface and base layer. Typical results for main shock and several aftershocks of EQ1 at 2 sites are exemplified for the EW direction in Figs. 2(a) and (b). There is a clear difference in the spectrum ratio between main shock and aftershocks in both sites reflecting the effects of the strain-dependent soil properties which result in nonlinear site response to strong shaking. In order to specify soil layers generating peak frequencies in the spectrum ratios, fundamental mode frequencies of layered soil systems fi were calculated by the following formula based on soil logging data along with S-wave velocities of individual layers.

\[
f_i = \left[ 4 \sum \left( H_i / V_{s_i} \right) \right]^{-1/2}
\]  

(1)

Here, Hi and Vs_i are the thickness and the S-wave velocity of the i-th layer numbered from the top, and the summation is implemented layer by layer down to the base. This frequency corresponds to a wave length which is equal to 4 times the layer thickness. The calculated frequency is compared one by one with the peak frequency in the spectrum ratio of observed motions such as in Figs. 2(a) and (b) to identify equivalent surface layers of thickness \( H' = \sum H_i \) consisting of one or more layers generating the
fundamental mode frequency calculated by Eq.(1). Note that there can be multiple equivalent surface layers in the same site corresponding to individual peak frequencies.

In Fig. 3, the peak frequencies $f_1$ calculated by Eq.(1) based on given soil data are taken in the horizontal axis to compare with the frequencies $f^*$ identified in the observed spectrum ratios for the main shock and aftershocks in the vertical axis for the recording sites of EQ3. It indicates that there exists a satisfactory correspondence between the fundamental mode frequency of soil models $f_1$ based on Eq.(1) and the peak frequency $f^*$ observed in spectrum ratios irrespective of the order of peaks. The average S-wave velocity $\bar{V}_s$ for each equivalent surface layer can be calculated from the fundamental mode frequency $f_1$ and its thickness $H_i = \sum H_i$ as

$$\bar{V}_s = 4H_i f_1$$

(2)

SPECTRUM RATIO BETWEEN SURFACE AND BASE

In seismic zonation of an area resting on a common base layer, a transfer function between ground surface and an outcropping base layer another transfer function $2A_s/(A_b + B_b)$ is obtained. Hence, the problem is how to evaluate $2A_s/(A_b + B_b)$ based on measured motions in the vertical arrays. A procedure chosen here is as follows (Kokusho and Sato 2008).

1) A theoretical transfer function, $2A_s/(A_b + B_b)$ is calculated for each site using given soil properties with frequency-independent damping ratio, $D$, assumed as, tentatively assumed as 2.5% in all layers.

2) The theoretical transfer function $2A_s/(A_b + B_b)$ is compared with measured spectrum ratio as exemplified in Fig. 4. If a peak in the transfer function can be found at about the same
frequency in the spectrum ratio of observed motions, it is identified as the corresponding peak, and the damping ratio, assumed as $D=2.5\%$ previously, is modified by $D = Q_1/Q_2 \times 2.5\%$ to have the same peak value, where $Q_1$ is the peak amplitude of the theoretical transfer function, and $Q_2$ is that of spectrum ratio based on the actual records. The calculation in EW and NS directions are carried out, and their average is taken.

3) Another theoretical transfer function $2A_s/2A_b$ is computed using the modified damping ratio $D$ based on the same soil layers model. When the two functions are not so similar as in the case in Fig. 4, peak frequencies of $2A_s/2A_b$ are compared with fundamental mode frequencies calculated by Eq.(1) and equivalent surface layers, and associated average S-wave velocities by Eq.(2) are determined again. The peak values of $2A_s/2A_b$ are adjusted using $D$-values determined from the nearest peaks of $2A_s/(A_b + B_b)$. In cases where such peaks are not found at all, that particular data is omitted.

S-WAVE VELOCITY RATIO VERSUS AMPLIFICATION FOR ZONATION

In the top of Fig. 5, the peak amplifications for $2A_s/2A_b$ computed by the methodology mentioned above for vertical array sites of EQ1 are plotted versus average S-wave velocity ratios $vs_b/\bar{v}_s$ for the main shock with solid symbols. The velocity ratio $vs_b/\bar{v}_s$ is defined here as a division of the S-wave velocity at a base layer $vs_b$ by the velocity $\bar{v}_s$ evaluated by Eq.(2) from the fundamental mode frequencies of Eq.(1) based on S-wave logging data at each site. A clear correlation can be recognized between the peak amplification and the velocity ratio both for 1st and higher order peaks. Four aftershock records of EQ1 are also analyzed in the same way and the results are plotted in the same graph with an open symbol.

A similar chart of the peak amplification for $2A_s/(A_b + B_b)$ plotted versus the same velocity ratio $vs_b/\bar{v}_s$ is shown in the bottom of Fig. 5. The difference in amplification for $2A_s/(A_b + B_b)$ between main shock and aftershocks is evidently larger than that for $2A_s/2A_b$. A simple 2-layers system of a surface layer and an infinitely thick base layer was studied to explain this difference (Kokusho and Sato 2008), which indicates that nonlinear properties make a great difference in the peak frequencies, though the difference in

Fig. 5 Peak amplitudes of $2A_s/A_b$ (top) and $2A_s/(A_b + B_b)$ (bottom) for main shock and aftershocks of EQ1 versus average S-wave velocity ratios.
the peak amplifications is less pronounced in $2A_s/2A_b$ than in $2A_s/(A_b + B_b)$ for the 1st peak in particular. That is because, under the paramount effect of radiation damping, the difference in the amplification $2A_s/2A_b$ due to strain-dependent properties becomes less conspicuous. Furthermore, the impedance ratio, which becomes smaller with degraded modulus or degraded S-wave velocity in the surface layer, tends to give larger amplification, compensating the effect of increased damping ratio during strong earthquakes.

Fig. 6 shows the relationship between the peak amplitude corresponding to $2A_s/2A_b$ and the velocity ratio $V_{S_b}/V_S$ plotted for main shocks of the eight earthquakes incorporated here. Totally 133 data points (95 for the 1st peak and 38 for the higher order peaks) from 37 recording sites are included in this chart (14 data points in which $V_{S_b}/V_S \geq 12$ are excluded here), and a large number of plots are overlapping in the zone of $V_{S_b}/V_S \leq 4.0$. Quite remarkably, the plots show a fairly good correlation including both 1st and higher order peaks despite differences in various influencing factors associated with the eight earthquakes; namely, dominant frequency, shaking duration, regional geological difference, etc.

In the current practice of seismic zonation, the S-wave velocity ratio defined as $V_{S_b}/V_{S_{30}}$ is sometimes used (Joyner and Fumal 1984, Midorikawa 1987) instead of $V_{S_b}/V_S$ proposed here, where $V_{S_{30}}$ = averaged S-wave velocity over top 30 m from ground surface. In Fig. 7, the same peak amplitudes corresponding to $2A_s/2A_b$ are plotted versus the velocity ratio, $V_{S_b}/V_{S_{30}}$. Here, $V_{S_{30}}$ (m/s) is calculated by the equation; $V_{S_{30}} = 30/T_{30}$, in which $T_{30}$ (s) is the traveling time of S-wave in the top 30 m based on soil logging data. Obviously, the correlation becomes poorer than in Fig. 6. Inconsistency in plots between 1st and higher order peaks is also evident. This indicates the importance to define the average S-wave velocity properly by identifying site by site the equivalent surface layer in which individual peak amplifications are exerted.

![Fig. 6](image-url)  
Fig. 6  Peak amplitudes of $2A_s/2A_b$ for main shocks of 8 earthquakes versus average S-wave velocity ratios $V_{S_b}/V_S$ proposed here and approximation by empirical equation.

![Fig. 7](image-url)  
Fig. 7  Peak amplitudes of $2A_s/2A_b$ for main shocks of 8 earthquakes versus average S-wave velocity ratios $V_{S_b}/V_{S_{30}}$ for surface soils of top 30m
In the previous research (Kokusho and Sato 2008), a simple linear function;

\[ 2A_s/2A_b = 0.175 + 0.685 \left( V_{s_b}/\sqrt{V_s} \right) \] (3)

was proposed for the condition of \( V_{s_b}/\sqrt{V_s} \leq 10 \), which also seems quite consistent with the data of the eight earthquakes. If the constants in Eq.(3) is newly computed from the data points in this paper, they slightly shift from 0.175 to 0.345 and 0.685 to 0.634. 

Eq.(3) may be conveniently used because of its applicability to a wide variety of base layers with \( V_{s_b} = 400 \text{ m/s to } 3000 \text{ m/s} \). The relative amplification for the same seismic motion in an area overlying a common base layer is readily evaluated. The procedure is as follows (Kokusho and Sato 2008):

1) Based on by microtremor measurements, decide fundamental frequency \( f_1 \) of a site using H/V spectrum ratios (Nakamura 1989). Estimate the thickness of a soft soil or Holocene layer \( H \) where the fundamental frequency is exerted, which is sometimes possible based on geological maps or high-density soil logging data available in city areas. Then, the average S-wave velocity can be calculated by \( \bar{V}_s = 4f_1H \).

2) Calculate the S-wave velocity ratio \( V_{s_b}/\bar{V}_s \) from \( V_{s_b} \) of the common base layer and \( \bar{V}_s \) obtained above for individual site conditions.

3) Comparing the amplifications by Eq.(3) at two different sites gives the relative amplification between them. To be precise, the amplification by Eq.(3) is slightly changeable depending on the value of \( V_{s_b} \) to be chosen among different base layers in the two sites, though its effect is ignorable for design purposes.

In the above, seismic response of ground is evaluated based on linear transfer function assuming that soil nonlinearity exerted during earthquakes is not so considerable. This assumption may not hold in those sites, such as Port Island during the 1995 Kobe earthquake (Kokusho et al. 2005). Strong soil nonlinearity by extensive liquefaction occurred there may have completely changed the soil system not to be able to justify the methodology employed here.

CONCLUSIONS

In studying seismic amplification between ground surface and base layer using KiK-net records of recent 8 strong earthquakes occurred in Japan, average S-wave velocity \( \bar{V}_s \) for equivalent surface layer corresponding to each peak of Fourier spectrum ratio was introduced from S-wave logging data. Then, a velocity ratio \( V_{s_b}/\bar{V}_s \) was defined by dividing S-wave velocity at a base layer \( V_{s_b} \) by the average velocity \( \bar{V}_s \) in the equivalent surface layer. The peak amplifications of the spectrum ratios were calculated and correlated with the velocity ratio. The spectrum amplifications relative to outcrop motions to be used for seismic zonation of surface ground resting on common base layer were computed from theoretical transfer functions and adjusted to be consistent with the peak amplifications of the array records.

The major findings in this series of research using 3 earthquakes and 8 earthquakes are;

1) The spectrum peak amplifications of \( 2A_s/2A_b \) for outcrop motions plotted versus the velocity ratios \( V_{s_b}/\bar{V}_s \) show a good correlation with a small data dispersion between the peak amplification and the velocity ratio both for 1st and higher order peaks, despite differences in influencing factors of individual earthquakes.
2) Strain-dependent soil nonlinearity tends to have a minimal effect on the peak amplifications of \( \frac{2A_s}{A_b} \) for surface arrays compared to those of \( \frac{2A_s}{(A_b + B)} \) for vertical arrays.

3) The correlation mentioned above obtained by utilizing 8 strong earthquakes coincides well with Eq.(3), which was previously proposed. Eq.(3) may be conveniently used for evaluating relative amplification in seismic zonation study covering an area sharing a common base layer with S-wave velocity, \( V_{sb} \).

4) If the same peak amplifications are plotted versus velocity ratios conventionally defined as \( \frac{V_{sb}}{V_{s0}} \) (\( V_{s0} = \) average velocity for top 30m sometimes used in the current seismic zonation practice), the correlation becomes poorer, indicating the importance to define the average S-wave velocity adequately by identifying a site-specific equivalent surface layer in which peak amplifications are exerted.

5) The velocity \( \bar{V_s} \) of the equivalent surface layer can be evaluated from \( V_s \) logging data, or if it is unavailable, can be decided from fundamental mode frequency \( f_1 \) of a site using H/V spectrum ratios in micro-tremor measurements together with the thickness of soft soil or Holocene layer \( H \) by \( \bar{V_s} = 4Hf_1 \).

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REFERENCES


