SEISMIC SITE AMPLIFICATION BASED ON VERTICAL ARRAY RECORDS AND SOIL NONLINEARITY EFFECT

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ABSTRACT

Site amplification formula was developed for virtual surface arrays to be consistent with the vertical array records using a number of KiK-net records during recent 8 destructive earthquakes. A correlation between peak spectrum amplification and S-wave velocity ratio (base/surface) was improved much better if the surface $V_s$ was evaluated from fundamental mode frequency and a thickness of equivalent surface layer in which peak amplification was exerted, rather than using the conventional $V_{s30}$. Also found was that soil nonlinearity effect during strong earthquakes was found to have insignificant effect on the amplification formula for surface array, even for the Kobe Port Island data, where extensive liquefaction took place.

Keywords: Site amplification, Vertical array, Vs-ratio, Soil nonlinearity

INTRODUCTION

Site amplification defined here as Fourier spectrum peak value between ground surface and bedrock depends on several factors; the composition of soil layers, S-wave velocities, soil densities, internal damping of the individual layers. Furthermore, strain-dependent nonlinear properties may affect the amplification in soft soils during strong earthquakes. Among the influencing factors, a ratio of S-wave velocity between the base layer and surface soil is of utmost importance. Shima (1978) found that the site amplification based on multi-reflection theory of SH wave is almost linearly related to the S-wave velocity ratio (base layer to surface top layer) despite the difference in intermediate soil layers.

In order to evaluate site amplification from ground depth to surface, NIED (National Research Institute for Earth Science and Disaster Prevention, Japan) deployed several hundreds of vertical array strong motion recording systems called KiK-net all over Japan, which comprises a pair of 3D accelerometers at ground surface and base layer. The observed records together with associated geotechnical data are easily accessible by international researchers at the website; http://www.kik.bosai.go.jp/kik/.

In general, there can be two different seismic array systems to measure the site amplification between ground surface and

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Figure 1: Surface array (a) and vertical array (b) for earthquake observations.
base layer as illustrated in Figs. 1(a) and (b). One is a surface array (a) consisting of a set of surface seismometers on different surface geologies in a relatively small area with a common base layer. Seismic records $2A_s$ on a surface soil and $2A_b$ on outcropping base layer allow to directly evaluate site amplification between the two different geologies $2A_s/2A_b$ if incident seismic wave $A_b$ is assumed basically the same in that area. The other is a vertical array (b) consisting of surface and down-hole seismometers at the same place. This can evaluate site amplification exactly at the same location, though some modification is necessary to extract the outcrop motion $2A_b$ from observed base motion $(A_b+B_b)$ which is more or less contaminated by downward wave $B_b$ from overlying layers.

In the context, Kokusho and Sato (2008) evaluated spectrum amplifications from strong motion records obtained by the KiK-net vertical arrays during 3 strong earthquakes occurred in recent years in Japan. The Fourier spectrum ratio $2A_s/(A_s+B_b)$ between surface and base was used to calculate the theoretical transfer function $2A_s/2A_b$ using multiple reflection theory of 1-dimensional SH waves by removing the effect of downward wave $B_b$ on the measured motion at the base layer. The peak amplifications of the transfer functions $2A_s/2A_b$ were then correlated with S-wave velocity ratios between base and surface which was newly proposed in the research.

In this paper, the number of earthquakes is increased from 3 to 8 to investigate if their spectrum peak amplifications are correlated in the similar manner with the S-wave velocity ratios. In addition, soil nonlinear effect on the amplification is investigated by comparing main shock records and those of small shocks obtained at the same KiK-net sites and also at the Port Island where extensive liquefaction occurred during the 1995 Kobe earthquake.

**EARTHQUAKE RECORDS AND AVERAGE V$_s$**

KiK-net records corresponding to 8 earthquakes (EQ1 to EQ8) listed in Table 1 is investigated in this research. The PGA-values in the vertical axis span from 100 to 2500 cm/s$^2$, while maximum horizontal accelerations at the base are less than 2000 cm/s$^2$, showing approximately 2-10 times amplification of the peak horizontal accelerations.

The depths of the down-hole seismometers in the vertical arrays used here vary from 100 m to 330 m except for 1 site (800 m). S-wave velocities in the top and base layers of all the vertical arrays are plotted in the vertical and horizontal axes in Fig. 2. S-wave velocity in the depth $V_{S_b}$ is normally stable.

![Figure 2 S-wave velocity $V_s$ at the top and at the base layers.](image)

Table 1  Earthquakes & KiK-net records used in this research

<table>
<thead>
<tr>
<th>EQ No.</th>
<th>Name of EQ</th>
<th>Year</th>
<th>JMA Mag. $M_J$</th>
<th>NO. of records</th>
<th>PGA (gal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tottoriken-Seibu</td>
<td>2000</td>
<td>7.3</td>
<td>20</td>
<td>$&gt;100$</td>
</tr>
<tr>
<td>2</td>
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<td>2001</td>
<td>6.4</td>
<td>13</td>
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</tr>
<tr>
<td>3</td>
<td>Tokachi-Oki</td>
<td>2003</td>
<td>8.0</td>
<td>19</td>
<td>$&gt;200$</td>
</tr>
<tr>
<td>4</td>
<td>Niigataken-Chuetsu</td>
<td>2004</td>
<td>6.8</td>
<td>15</td>
<td>$&gt;100$</td>
</tr>
<tr>
<td>5</td>
<td>Fukuokaen-Seiho-Oki</td>
<td>2005</td>
<td>7.0</td>
<td>10</td>
<td>$&gt;100$</td>
</tr>
<tr>
<td>6</td>
<td>Noto-Hanto</td>
<td>2007</td>
<td>6.9</td>
<td>7</td>
<td>$&gt;100$</td>
</tr>
<tr>
<td>7</td>
<td>Niigataken-Chuetsu-Oki</td>
<td>2007</td>
<td>6.8</td>
<td>9</td>
<td>$&gt;100$</td>
</tr>
<tr>
<td>8</td>
<td>Iwate-Miyagi Nairiku</td>
<td>2008</td>
<td>7.2</td>
<td>11</td>
<td>$&gt;150$</td>
</tr>
</tbody>
</table>
ineach site and does not drastically change among neighboring layers. However, the $V_S$ values among different sites diverge from $V_S = 400$ m/s to 3000 m/s in the areas of the eight earthquakes as plotted in Fig. 2. In a good contrast, the surface velocities $V_S$ are around 200 m/s on average for most of the recording sites used here.

Fourier spectrum ratio $2A_i/(A_b + B_b)$ between surface and base is computed between ground surface and base layer for each vertical array site. A typical result for main shock and several aftershocks at a site during EQ3 is exemplified for the EW direction in Figs. 3(a) and (b). There is a clear difference in the spectrum ratio between the main shock and the aftershocks reflecting the effects of the strain-dependent soil properties which result in nonlinear site response to strong shaking. In order to specify soil layers generating peak frequencies in the spectrum ratio, fundamental mode frequencies of layered soil systems $f$ were calculated by the following 1/4-wave length formula based on soil logging data along with S-wave velocities of individual layers.

$$f = \frac{1}{2\pi} \sqrt{\frac{\sum H_i}{\sum V_S}}$$  \hspace{1cm} (1)

Here, $H_i$ and $V_S$ are the thickness and the S-wave velocity of the $i$-th layer numbered from the top, and the summation is implemented layer by layer down to the base. This frequency $f$ corresponds to the layer thickness which is equal to 1/4 wave length. The calculated frequency is compared one by one with the peak frequency in the spectrum ratio of observed motions such as in Fig. 3(a) to identify equivalent surface layers of thickness $H_s = \sum H_i$ consisting of one or more layers generating the fundamental mode frequency calculated by Eq.(1) as tabulated in Fig. 3(b). Note that there can be multiple equivalent surface layers in the same site corresponding to individual peak frequencies.

In Fig. 4, the peak frequencies $f$ calculated by Eq.(1) based on given soil data are taken in the horizontal axis to compare with the frequencies $f^*$ identified in the observed spectrum ratios for the main shock records of the 8 earthquakes. It indicates that there exists a satisfactory correspondence

![Figure 3 Typical spectrum ratios for a main shock and aftershocks during EQ3 (a) and peak frequencies calculated by 1/4 wave length formula (b).](image)

![Figure 4 Peak frequencies calculated by Eq.(1) compared with identified frequencies in observed spectrum ratios for main shock records of 8 earthquakes.](image)
between the fundamental mode frequency of soil models $f$ based on Eq.(1) and the peak frequency $f^*$ observed in spectrum ratios irrespective of the order of peaks. The average S-wave velocity $V_S$ for each equivalent surface layer can be calculated from the fundamental mode frequency $f$ and its thickness $H_i = \sum H_i$ as

$$V_S = 4H_i f$$  \hspace{1cm} (2)

**SURFACE/BASE SPECTRUM RATIO**

In seismic zonation of an area resting on a common base layer, a transfer function between ground surface and an outcropping base layer $2A_s/2A_b$ is needed, while in vertical array records another transfer function $2A_s/(A_b + B_b)$ is obtained as explained in Fig.1. Hence, the problem is how to evaluate $2A_s/2A_b$ based on measured motions in the vertical arrays. A procedure chosen here is as follows (Kokusho and Sato 2008).

1) A theoretical transfer function, $2A_s/(A_b + B_b)$ is calculated for each site using given soil properties with frequency-independent damping ratio, $D$, tentatively assumed as 2.5% in all layers.

2) The function $2A_s/(A_b + B_b)$ is compared with measured spectrum ratio as exemplified in Fig. 5. If a peak in the transfer function can be found at about the same frequency in the spectrum ratio of observed motions, it is identified as the corresponding peak, and the damping ratio, assumed as $D=2.5\%$ previously, is modified by $D = Q_1/Q_2 \times 2.5\%$ to have the same peak value, where $Q_1$ is the peak amplitude of the theoretical transfer function, and $Q_2$ is that of spectrum ratio of the actual records. The values of $D$ calculated in EW and NS directions are averaged.

3) Another theoretical transfer function $2A_s/2A_b$ is computed using the modified damping ratio $D$ based on the same soil layers model. When the peak frequencies of the two functions are different unlike the case in Fig. 5, peak frequencies of $2A_s/2A_b$ are compared with fundamental mode frequencies calculated by Eq.(1) and equivalent surface layers, and associated average S-wave velocities by Eq.(2) are determined again. The peak values of $2A_s/2A_b$ are adjusted using $D$-values determined from the nearest peaks of $2A_s/(A_b + B_b)$. When the two transfer functions were very different with completely different peak frequencies, that particular site was not used for the data analysis.

**PEAK AMPLIFICATION VERSUS VS-RATIO**

In the current practice of seismic zonation, the average S-wave velocity $V_{S30}$ is sometime used in making simple evaluation of site amplification (Joyner and Fumal 1984, Midorikawa 1987), where $V_{S30} = \text{averaged S-wave velocity over top } 30\text{ m from ground surface}$. In Fig. 6, the peak values in $2A_s/2A_b$ are plotted versus the velocity ratio, $V_{S_b}/V_{S30}$, for all main shock records obtained during the 8 strong earthquakes. Here, $V_{S_b} = \text{S-wave velocity at a base layer}$, and $V_{S30} = \text{S-wave velocity in the layer of top}$
30m, calculated by the equation; \( V_{S30} = 30/T_{30} \) in which \( T_{30} \) (s) is the travel time of S-wave in the soil of top 30 m. The total number of plots is 134 from 102 recording sites (95 for the 1st peak and 39 for the higher order peak), among that 4 plots with the peak amplification larger than 8 are outside the chart. Obviously, a positive correlation can be recognized, though data are largely scattered. Inconsistency is evident in plots among different earthquakes and 1st and higher order peaks.

Fig. 7 shows the relationship between the peak amplitude corresponding to \( 2A_s/2A_b \) and the \( Vs - ratio \) plotted for main shocks of the eight strong earthquakes. The \( Vs - ratio \), \( V_{Sb}/Vs \), is defined here as a division of the S-wave velocity at a base layer \( V_{Sb} \) by the velocity \( Vs \) evaluated by Eq.(2) from the fundamental mode frequencies of Eq.(1) based on S-wave logging data at each site. Totally 131 data points from 95 recording sites are included in this chart (93 for the 1st peak and 38 for the higher order peak, and 16 data points in which \( V_{Sb}/Vs \geq 10 \) are outside the chart), and a large number of plots are overlapping in the zone of \( V_{Sb}/Vs \leq 4.0 \). In a good contrast to Fig. 6 where \( V_{S30} \) is used instead of \( Vs \), the plots in Fig. 7 show a fairly good correlation including both 1st and higher order peaks despite differences in various influencing factors associated with the eight earthquakes; namely, dominant frequency, shaking duration, regional geology, etc.

This indicates the importance to define the average
S-wave velocity properly by identifying site by site the equivalent surface layer in which individual peak amplifications are exerted.

In the previous research (Kokusho and Sato 2008), a simple linear function;

\[
2A_s/(2A_b + B_b) = 0.175 + 0.685\left(\frac{V_{S_b}}{V_S}\right)
\]

(3)

was proposed based on 3 earthquakes (EQ3, EQ4 and EQ5) for the condition of \(V_{S_b}/V_S \leq 10\), which also seems quite consistent with the data of the eight earthquakes. If the constants in Eq.(3) is newly computed from all the data points in this paper, the constants in Eq.(3) slightly shift from 0.175 to 0.369 and 0.685 to 0.626, and the regression coefficient changes from 0.93 to 0.89.

Eq.(3) or its modified version may be conveniently used because of its applicability to a wide variety of base layers with \(V_{S_b}=400 \text{ m/s} \) to 3000 m/s. The relative amplification for the same seismic motion in an area overlying a common base layer is readily evaluated. The procedure is as follows (Kokusho and Sato 2008);

1) Based on by microtremor measurements, decide fundamental frequency \(f\) of a site using H/V spectrum ratios (Nakamura 1989). Estimate the thickness of a soft soil or Holocene layer \(H\) where the fundamental frequency is exerted, which is sometimes possible based on geological maps or high-density soil logging data available in city areas. Then, the average S-wave velocity can be calculated by \(V_S = 4Hf\).

2) Calculate the S-wave velocity ratio \(V_{S_b}/V_S\) from \(V_{S_b}\) of the common base layer and \(V_S\) obtained above for individual site conditions.

3) Comparing the amplifications by Eq.(3) at two different sites gives the relative amplification between them. To be precise, the amplification by Eq.(3) is slightly changeable depending on the value of \(V_{S_b}\) to be chosen among different base layers in the two sites, though its effect is ignorable for design purposes.

**SOIL NONLINEARITY EFFECT ON PEAK AMPLIFICATION VERSUS \(\overline{V_S}\)-RATIO**

In Figs. 8(a) and (b), the peak amplifications for the two kinds of spectrum ratio, \(2A_s/(A_b+B_b)\) (a) and \(2A_s/2A_b\) (b), versus \(V_{S_b}/\overline{V_S}\) are compared for EQ3. The solid and open symbols are for the main shock and the aftershocks, respectively, and 1st and higher order peaks are also differentiated with different symbols. A clear correlation can be recognized between the peak amplification and the velocity ratio of the main shock both for the peaks of \(2A_s/(A_b+B_b)\) in (a) as well as of \(2A_s/2A_b\) in (b). Plots for the 4 aftershocks are located far from that of corresponding main shock in (a) particularly for higher order

![Figure 8](https://example.com/figure8.png)

Figure 8 Peak amplifications of \(2A_s/(A_b+B_b)\) (left) and \(2A_s/2A_b\) (right) for main shock and aftershocks of EQ3 versus average S-wave velocity ratios \(V_{S_b}/\overline{V_S}\)
peaks, while they are much closer to each other in (b). In other words, the difference in amplification for \(2A_s/(A_0 + B_b)\) between main shock and aftershocks is evidently greater than that for \(2A_s/2A_b\), indicating that the soil nonlinearity effect seems less conspicuous in \(2A_s/2A_b\) than in \(2A_s/(A_0 + B_b)\).

In order to account for this difference, a simple 2-layers system of a surface layer (Impedance= \(\rho V_{s1}\), Damping ratio=\(D_1\)) and an infinitely thick base layer (Impedance= \(\rho V_{s2}\), Damping ratio=\(D_2\)) shown in Fig. 9(a) was studied (Kokusho and Sato 2008), assuming the impedance ratio for small strain properties as \(\alpha = \rho V_{s1}/\rho V_{s2} = 0.3\). The transfer functions \(2A_s/(A_0 + B_b)\) and \(2A_s/2A_b\) can be expressed by the following equations, respectively:

\[
2A_s/(A_0 + B_b) = 2\left( e^{ik_1 H} + e^{-ik_1 H} \right)
\]

(4)

\[
2A_s/2A_b = 2\left( (1 + \alpha^2)e^{ik_1 H} + (1 - \alpha^2)e^{-ik_1 H} \right)
\]

(5)

where \(k_1 = \sqrt{\nu_1^2 + 2\nu D_1}\) and \(\alpha^2 = \sqrt{1 + 2D_1}/(\nu_2 + 2D)\).

In taking account the effect of strain-dependent soil properties on the amplification, the shear modulus ratio \(G/G_0\) and the damping ratio \(D_1\) in the surface layer are parametrically changed; \(G/G_0 = 1.0, 0.65, 0.25\) and \(D_1 = 2.5, 5\) and 15% correspondingly, for strain level of \(5 \times 10^{-6}, 1 \times 10^{-4}, 1 \times 10^{-3}\); respectively, assuming a typical degradation curve for sand (Seed and Idriss 1970) as indicated in Fig.9(b), while in the base layer \(D_2 = 0\). The calculated results of \(2A_s/(A_0 + B_b)\) and \(2A_s/2A_b\) are shown in Figs. 9(c) and (d), respectively, in which the amplification between surface and base is taken versus normalized frequency, \(f/f_1\), where \(f_1 = \)fundamental mode frequency of the surface layer for small

![Figure 9](image-url)

Figure 9  Two-layers system (a), Strain-dependent properties of surface layer (b), \(2A_s/(A_0 + B_b)\) (c) and \(2A_s/2A_b\) (d); for different strain levels in surface layer.
strain properties \((G/G_0 = 1.0)\).

Obviously, nonlinear soil properties have great effects on the peak frequencies and the amplifications. However, the difference in the peak amplifications is less pronounced in \(2A_s/2A_b\) than in \(2A_s/(A_b + B_b)\) for the 1st peak in particular, because radiation damping effect represented by impedance ratio \(\alpha^*\) affects \(2A_s/2A_b\) in Eq.(5), whereas no effect of impedance ratio \(\alpha^*\) is involved in \(2A_s/(A_b + B_b)\) as indicated in Eq.(4). Under the paramount effect of radiation damping, the difference in the amplification \(2A_s/2A_b\) due to strain-dependent properties becomes less dominant. Furthermore, the impedance ratio \(\alpha = \rho V_s/\rho_0 V_{s0}\), which becomes smaller with degraded modulus or degraded S-wave velocity in the surface layer, tends to give larger amplification compensating the effect of increased damping ratio during strong earthquakes. Thus, the difference in soil properties between the main shock and aftershocks tends to have smaller influence on the amplification in \(2A_s/2A_b\) than in \(2A_s/(A_b + B_b)\) as demonstrated in the comparison of Figs. 8(a) and (b).

In the above, seismic response of ground was evaluated based on linear transfer function assuming that soil nonlinearity exerted during earthquakes is not so considerable. Actually no KiK-net records studied here appears to have demonstrated strong soil nonlinearity. Hence, Port Island (PI) vertical array records during the 1995 Kobe earthquake, much influenced by extensive liquefaction in surface fill layer of about 15 m thick, are examined in the same methodology as before in the following.

In Figs. 10 (a) and (b), transfer function \(2A_s/(A_b + B_b)\) of PI calculated between surface and two down-hole levels, GL-32.4 m and GL-83.4 m, using small-strain \(V_s\) by wave logging, are compared with corresponding spectrum ratios calculated from small shock motions recorded before the main shock (Kokusho et al. 2005). In Fig. 11, the peak amplifications of \(2A_s/2A_b\) calculated in the same way as for the KiK-net records are plotted versus the \(V_s\)-ratios based on small strain \(V_s\) with the open

![Figure 10](image-url)

**Figure 10** Transfer functions of Port Island (PI) vertical array; (a),(b): based on logging \(V_s\) compared with observed spectrum ratios of small shocks, (c)(d): based on back-calculated \(V_s\) for main shock compared with observed spectrum ratios of main shock.
circle and the open triangle. These plots are located near the straight line of Eq.(3), indicating satisfactory evaluation of amplification by the proposed formula between ground surface and two different depths. In Figs. 10 (c) and (d), transfer functions calculated for back-calculated Vs (Kokusho et al. 2005) for the main shock are compared with observed spectrum ratios for the main shock. The peak amplifications are plotted versus the $\overline{Vs}$-ratios based on back-calculated Vs with the corresponding solid symbols. The plots are evidently lower than the proposed formula presumably due to higher damping ratio associated with extensive soil liquefaction.

In obtaining Fig.7 using KiK-net records, small-strain Vs was used to compute values of $\overline{Vs}$ and $\overline{Vs}$-ratios to correlate with peak amplifications of main shocks as mentioned before. If the same procedure is taken for the PI records, the $\overline{Vs}$-ratios for small strain Vs are to be correlated with the main shock peak values. The plots thus obtained are shown on Fig.11 and also on Fig.7 with open star symbols. This indicates that even strongly nonlinear amplification in PI reflecting extensive liquefaction may be roughly approximated by the proposed formula Eq.(3).

CONCLUSIONS

Site amplification formula for seismic zoning was developed to be consistent with the vertical array records using a number of KiK-net records during recent 8 destructive earthquakes. Average S-wave velocity $\overline{Vs}$ for an equivalent surface layer, generating a peak in observed spectrum ratio between surface and a base layer, was introduced to be correlated to the calculated peak amplification of outcrop motions, giving the following findings:

1) The spectrum peak amplifications of $2A_s/2A_b$ for outcrop motions plotted versus the $\overline{Vs}$-ratios ($v_{sb}/\overline{Vs} : v_{sb} = Vs$ at a base layer) show a good correlation with a small data dispersion between the peak amplification and the velocity ratio both for 1st and higher order peaks, despite differences in influencing factors of individual earthquakes.

2) The correlation mentioned above obtained by utilizing 8 strong earthquakes coincides well with Eq.(3), which was previously proposed. Eq.(3) or its modified version in the present paper may be conveniently used for evaluating relative amplification in seismic zonation study covering an area sharing a common base layer with S-wave velocity, $v_{sb}$.

3) If the same peak amplifications are plotted versus velocity ratios defined by $v_{sb}/v_{s30}$ ($v_{s30} =$ average velocity for top 30m sometimes used in the current seismic zonation practice), the correlation becomes poorer, indicating the importance to define the average S-wave velocity adequately by identifying a site-specific equivalent surface layer in which peak amplifications are exerted.

4) The velocity $\overline{Vs}$ of the equivalent surface layer can be evaluated from Vs-logging data, or if it is unavailable, may be determined from fundamental mode frequency $f$ of a site using H/V spectrum ratios in micro-tremor measurements together with the thickness of soft soil or Holocene layer $H$ by $\overline{Vs} = 4hf$.

Furthermore, the effect of nonlinear soil properties was studied by comparing peak amplifications for
main shock and small shocks recorded at the same sites, revealing the followings;
5) Strain-dependent soil nonlinearity has a minor effect on the peak amplifications of $2A_s/2A_b$ for surface arrays (outcrop motions) compared to those of $2A_s/(A_0 + B_0)$ for vertical arrays.
6) The logical bases of the minor nonlinear effect on $2A_s/2A_b$ can be explained by a simple 2-layers equivalent linear system considering strain-dependent soil nonlinearity.
7) Even the peak amplification of $2A_s/2A_b$ at PI during the 1995 Kobe earthquake where extensive liquefaction occurred may be roughly approximated by the proposed formula.

Acknowledgments

NIED (National Research Institute for Earth Science and Disaster Prevention) in Tsukuba, who disseminated numerous KiK-net data, and Kobe city, who provided Port Island vertical array data during the 1995 Kobe earthquake, are gratefully acknowledged. The great efforts in data reduction of voluminous vertical array records by graduate and undergraduate students of Civil Engineering Department in Chuo University are also very much appreciated.

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