Parameter Identification of Elastic Modulus of Rock Based on Blast Vibration by Adjoint Equation Method

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Abstract

This paper presents a parameter identification of the elastic modulus of the ground using the finite element method and optimal control theory. In this research, the elastic modulus is identified to minimize the performance function which consists of the square residual between observed and computed data, using the adjoint method, which is one of the techniques of the inverse analysis. The actual site is assumed as an elastic body. To calculate the velocity of wave transmitted through the ground, the equilibrium of stress equation, the strain-displacement equation, and the stress-strain equation are employed. To identify the elastic modulus, the gradient of the performance function is calculated as the direct form. The weighted gradient method is employed for the minimization algorithm. The Galerkin method and the Newmark β method are employed to discretize in space and temporal directions, respectively. The elastic modulus at the Futatsuishi quarry site has been determined by this analysis.

Key words : finite element method, parameter identification, adjoint equation method, elastic modulus
weighted gradient method

1 Introduction

Recently, rock-fill dams are constructed in environmental point of view, which piles up clay and rock and is built merging with a surrounding natural material. The clay zone stops water, and gravel zone prevented the collapse of clay zone. Moreover, the rock stabilizes the dam pressing the whole body. Thus, various kinds of rocks are used in the construction of dams. According to the construction plan of the dam, the rock-fill dam, needs particular hardness and quantity of rock. It is necessary to understand the ground characteristic as accurate as possible. The uselessness of the collection can be omitted by distinguishing CL and CM by the difference of the elastic modulus and understanding the ground properties. The loss in the weight of the abandonment rock should be minimized and, it is connected with cutting down the construction cost.

In case that construction is designed, the ground properties of the bedrock are usually investigated by the geologic survey of drilling, a physical inquiry (elastic wave inquiry and electric inquiry, etc.), and the rock test for engineering properties, etc. However, if the geologic structure is complex, it is difficult to understand the geological features and properties accurately. The purpose of this research is a prior forecast of the ground properties of the bedrock to improve the efficiency of construction.

In this paper, the finite element method and the adjoint method of the optimal theory are used by the observation velocity of the blast vibration wave as a numerical analysis technique for requesting the elastic modulus in the ground. In this research, the numerical model is set at the Futatsuishi quarry site in Miyagi Prefecture, Japan. The possibility of parameter identification based on the present method is confirmed by the application to the practical site.
2 Basic Equation

In this paper, indicial notation and summation convention with repeated indices are used. As the basic equation the following three equations are employed.

**Equilibrium of Stress Equation:**

\[ \sigma_{ij,j} - \rho b_i + \rho \ddot{u}_i = 0, \quad (1) \]

where \( \sigma_{ij} \) is total stress, \( \ddot{u}_i \) is acceleration of wave transmitted through the ground, \( \rho, b_i \), denote ground density and body force, respectively.

**Strain-Displacement Equation:**

\[ \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad (2) \]

where \( \varepsilon_{ij} \) and \( u_i \) are strain and displacement of wave transmitted in the ground, respectively.

**Stress-Strain Equation:**

\[ \sigma_{ij} = D_{ijkl} \varepsilon_{kl}, \quad (3) \]

where \( D_{ijkl} \) is elastic stress-strain tensor and can be written as follows:

\[ D_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}), \quad (4) \]

where \( \delta_{ij} \) is Kronecker’s delta, and Lamé’s constants \( \lambda, \mu \) are

\[ \lambda = \frac{\nu E}{(1-2\nu)(1+\nu)}, \quad (5) \]

\[ \mu = \frac{E}{2(1+\nu)}, \quad (6) \]

where \( E \) is elastic modulus and \( \nu \) is Poisson’s ratio, respectively.

Basic equations are solved on the following boundary conditions. The boundary \( \Gamma \) can be divided into \( \Gamma_1 \) and \( \Gamma_2 \), which are Dirichlet and Neumann boundaries, respectively. On these boundaries, the following conditions are specified.

\[ u_i = \hat{u}_i \quad \text{on} \ \Gamma_1, \quad (7) \]

\[ t_i = \sigma_{ij} n_j = \hat{t}_i \quad \text{on} \ \Gamma_2, \quad (8) \]

where \( \hat{u}_i, \hat{t}_i \) and \( n_i \) mean the known value on the boundaries of displacement and surface force and external unit vector to the boundary, respectively.

3 Finite Element Equation

Applying the finite element method, the discretization with the linear tetrahedral element is obtained as follows:

\[ M_{\alpha i\beta k} \ddot{u}_{\beta k} + K_{\alpha i\beta k} u_{\beta k} = \hat{\Gamma}_{\alpha i}, \quad (9) \]

Considering the effect of damping, eq.(9) can be expressed as;
\begin{align}
M_{\alpha\beta k} \ddot{u}_{\beta k} + C_{\alpha\beta k} \dot{u}_{\beta k} + K_{\alpha\beta k} u_{\beta k} = \Gamma_{\alpha i},
\end{align}

where coefficients are as;

\begin{align}
M_{\alpha i\beta k} &= \int_{\Omega} (\rho N_{\alpha} N_{\beta\delta k}) d\Omega, \\
K_{\alpha i\beta k} &= \int_{\Omega} (N_{\alpha,j} D_{ijkl} N_{\beta,l}) d\Omega, \\
C_{\alpha i\beta k} &= \alpha_0 M_{\alpha\beta} + \alpha_1 K_{\alpha i\beta k},
\end{align}

\begin{align}
\Gamma_{\alpha i} &= \int_{\Omega} (\rho N_{\alpha} b_i) d\Omega - \int_{\Gamma} (N_{\alpha} \dot{t}_i) d\Gamma,
\end{align}

in which \(N_{\alpha}\) is the linear shape function, and \(\alpha_0\) and \(\alpha_1\) are damping coefficients, respectively.

4 Newmark \(\beta\) method

In this paper, the Newmark \(\beta\) method is applied to the finite element equation. In the Newmark \(\beta\) method, velocity and displacement at \((n+1)\) time are

\begin{align}
\dot{u}_i^{(n+1)} &= \dot{u}_i^{(n)} + \frac{1}{2} \ddot{u}_i^{(n)} \Delta t + \beta \Delta t^2 (\ddot{u}_i^{(n+1)} - \ddot{u}_i^{(n+1)}), \\
\ddot{u}_i^{(n+1)} &= \ddot{u}_i^{(n)} + \gamma \Delta t (\ddot{u}_i^{(n+1)} - \ddot{u}_i^{(n+1)}),
\end{align}

where \(u_i^{(n+1)}\) and \(\dot{u}_i^{(n+1)}\) denote displacement and velocity, respectively. Parameter \(u_i^{(n+1)}\) and \(\dot{u}_i^{(n+1)}\) are substituted into eq.(10), the following equation can be derived where parameters \(\beta\) and \(\gamma\) are assumed as 0.25 and 0.50, respectively.

\begin{align}
[D][\ddot{u}^{(n+1)}] = \Gamma_{\alpha i} - [E][\dddot{u}^{(n)}] - [F][\ddot{u}^{(n)}] - [K][u^{(n)}],
\end{align}

\([D], [E], [F]\) can be written as,

\begin{align}
[D] &= [M] + \gamma [C] dt + \beta [K] dt^2, \\
[E] &= (1 - \gamma) [C] dt + \left(\frac{1}{2} - \beta\right) [K] dt^2, \\
[F] &= [C] + [K] dt,
\end{align}

acceleration at \((n+1)\) time is substituted into eqs.(15) and (16) to calculate \(u_i^{(n+1)}, \dot{u}_i^{(n+1)}\), where indice are omitted.
5 Performance Function

In this paper, the parameter identification problem is performed, which is defined as finding optimal elastic modulus so as to minimize the performance function $J$;

$$J = \frac{1}{2} \int_t Q_{ij}(\dot{u}_i - \dot{u}_i^*)(\dot{u}_j - \dot{u}_j) dt,$$

(21)

where $\dot{u}_i$ and $\dot{u}_i^*$ are the computed and the observed velocity, $Q_{ij}$ is weighting function, respectively. Namely, the problem pursued in this research is that the optimal elastic modulus of the ground is found as the computed velocity is as close as the observed velocity.

6 First Order Adjoint Method

In this research, the extended performance function consists of wave velocity, and the objective parameter is chosen as elastic modulus. The extended performance function $J^*$ is expressed as follows;

$$J^* = \frac{1}{2} \int_t Q_{ij}(\dot{u}_i - \dot{u}_i^*)(\dot{u}_j - \dot{u}_j) dt + \int_t \lambda_{ai}(F_{ai} - M_{ai\beta k}\ddot{u}_{\beta k} - C_{ai\beta k}\dot{u}_{\beta k} - K_{ai\beta k}u_{\beta k}) dt,$$

(22)

where $\lambda_{ai}$ is Lagrange multiplier. The first variation of the extended performance function $\delta J^*$ is expressed as follows;

$$\delta J^* = \int_t Q_{ij}(\ddot{u}_i - \ddot{u}_i^*)\delta u_j dt + \int_t \delta \lambda_{ai}(F_{ai} - M_{ai\beta k}\ddot{u}_{\beta k} - C_{ai\beta k}\dot{u}_{\beta k} - K_{ai\beta k}u_{\beta k}) dt$$

$$- \int_t \lambda_{ai}(M_{ai\beta k}\ddot{u}_{\beta k} + C_{ai\beta k}\dot{u}_{\beta k} + K_{ai\beta k}\delta u_{\beta k} + \alpha_1 (\dddot{u}_i^* + u_{\beta k}\delta E + K_{ai\beta k}\delta E)) dt.$$

$$= 0,$$

(23)

$$K_{ai\beta k}^* = \int_{\Omega} \left\{ N_{aij} \frac{\nu}{(1 - 2\nu)(1 + \nu)} \delta_{ij} \delta_{\beta \ell} + \frac{1}{2(1 + \nu)} \delta_{ik} N_{\beta j,\ell} \right\} d\Omega.$$

(24)

The adjoint equation can be obtained as;

$$M_{ai\beta k}\ddot{\lambda}_{ai} - C_{ai\beta k}\dot{\lambda}_{ai} + K_{ai\beta k}\lambda_{ai} + Q_{ij}(\dddot{u}_i - \dddot{u}_i^*) = 0.$$

(25)

The terminal conditions are needed to solve the adjoint equation. Those are;

$$\lambda_{ai}(t_f) = 0,$$

(26)

where $t_f$ is the terminal time and;

$$M_{ai\beta k}\ddot{\lambda}_{ai}(t_f) + Q_{ij}(\dddot{u}_j - \dddot{u}_j^*) = 0.$$

(27)

The terminal condition of acceleration is $\dddot{\lambda}_{ai}(t_f)$, which can be solved using $\dddot{\lambda}_{ai}(t_f)$ and $\lambda_{i}(t_f)$. Eqs.(28) to (30) are employed to solve the inverse analysis.

After solving the adjoint equation, eq(26) can be transformed into the following form.

$$\delta J^* = \int_t \lambda_{ai} K_{ai\beta k}^*(\alpha_1 \dddot{u}_{\beta k} + u_{\beta k}) E dt.$$

(28)

The gradient of the extended performance function can be calculated.

$$\text{grad}(J^*)_E = \lambda_{ai} K_{ai\beta k}^*(\alpha_1 \dddot{u}_{\beta k} + u_{\beta k}).$$

(30)

where $\text{grad}(J^*)_E$ is the gradient of the extended performance function with respect to $E$. The purpose of this research is to identify the elastic modulus. Thus, the gradient of extended performance function is used in minimization technique.
The Weighted Gradient Method

The gradient method is applied to obtain the minimization technique in this research. The modified performance function is

\[
K = J^* + \int (E^{(n+1)} - E^{(n)})^T W (E^{(n+1)} - E^{(n)}) \, dt,
\]

(31)

where \( W \) is the parameter and the weighting diagonal matrix. The second term is added to obtain the numerical stability. The optimality condition can be obtained as the particle differentiation of the extended performance function with respect to parameter.

\[
\frac{\partial K}{\partial E^{(n+1)}} = 0.
\]

(32)

The gradient of performance function is calculated by to obtain the parameter \( E \).

\[
E^{(n+1)} = E^{(n)} + \text{grad}(J^*) E/W,
\]

(33)

Using eq.(24), the elastic modulus is updated at each iteration cycle.

8 Futatsuishi Quarry Site

The Futatsuishi dam construction site is located in Miyagi prefecture in Japan. Close to the dam site, the quarry is set of which site map is shown in Fig.1. Fig.2 represents the finite element mesh of which total number of nodes and elements are 11614 and 60771, respectively. Roughly, the ground stratum at the site is classified into two classes, i.e., hard stratum, which is denoted by CM and soft stratum, which is by Cl as shown in Fig.3.

The Poisson’s ratio and density of ground is given as 0.3 and 2.3\([g/cm^3]\), respectively. Those are determined by other experimental investigation. As the boundary condition, bottom face is assumed as non-slip condition, which means three components of displacement are zero and surface of the ground is all stress free. For damping coefficient \( \alpha_0 \) and \( \alpha_1 \) are 0.005 and 0.00033 are used. For the reference data \( \dot{u}_i \), the observed velocity by the blast is used. Fig.3 shows the blast zone and observation points. In this study, two observation points are used. The blast impact is assumed as the equivalent distributed momentary external force, which is applied during a time increment. The external force at the blast zone is \( 1.7 \times 10^7[kN/m^2] \times 105[m^2] \), as shown in Fig.5. This value is obtained by the conventional formula of the blast impact and several iteration of computation. The length and width of the blast zone is assumed as \( 35[m] \times 3[m] \), as shown in Fig.3.

9 Numerical Study 1

In the numerical study 1, parameter identification presented in this paper is verified. The elastic moduli at CM and CL zone are also assumed as \( 3.0 \times 10^6[kN/m^2] \) and \( 8.0 \times 10^5[kN/m^2] \), respectively. Imposing the external force as shown in Fig.5 and using these elastic moduli, the computed velocity at the observation points can be obtained. These data are referred to as "the computed observation". The parameter identification is carried out using the computed observation and starting from the initial moduli at CM and CL zones as \( 3.0 \times 10^5[kN/m^2] \).

Finally, the elastic moduli at CM and CL zones can be computed as \( 3.0 \times 10^6[kN/m^2] \) and \( 8.0 \times 10^5[kN/m^2] \). Those are completely coincident with the target values. Thus, the present method is verified. Fig.6 represents the computational history of the elastic moduli, which are converged to the objective values at CM and CL zones. Fig.7 shows the computational history of the performance function, which is converged to zero. In Fig.8 computational observed velocity was compared with the computed velocity. Two lines are exactly corresponding. This also means the present method is verified. The time increment \( \Delta t \) used in the computation is 0.001[sec].
10 Numerical study 2

In the numerical study 2, parameter identification of elastic modulus by using observed velocity data at Futatsuishi site has been carried out. The unknown parameters are Poisson’s ratio, density of ground, the elastic modulus, damping coefficient and external force, respectively. Some of which were presumed based on the material tests at the quarry site. The Poisson’s ratio, density of ground, and external force, are assumed as 0.3, 2.3[g/cm$^3$] 1.7 × 10$^7$[kN/m$^2$], respectively. Because the damping coefficients were still unknown. Several forward analyze are carried out based on the assumed data, and comparison of the observed velocity at the site to determine the damping parameters.

Using these pre assumed parameters and observed velocity at two observation points shown in Fig.3, parameter identification of CL and CM zones is carried out. Fig.9 shows the history of the performance function. As shown in the figure, the performance function is converged. Fig.10 represents the history of the elastic modulus. The elastic modulus at each stratum is converged. Thus, the elastic moduli of CM and CL strata have been determined. The elastic modulus obtained at CM stratum is 9.8 × 10$^6$[kN/m$^2$], and at CL stratum is 3.9 × 10$^6$[kN/m$^2$], respectively. These value are compared with result of the elastic wave test by the Futatsuishi quarry site shown in Table-1. The elastic moduli of CL and CM zone are reasonably coincident with those obtained by the elastic wave test. Fig.11 represents the comparison of observed velocity data and computed using the velocity identified elastic moduli. Two data is almost the same wave form. Therefore, it is understood that the reasonable elastic moduli have been obtained.

11 Conclusion

In this research, the parameter identification as two stratum of the elastic modulus using the adjoint method is presented. The Futatsuishi quarry site is used as the computational model. The elastic modulus of each stratum is converged. The numerical results are well in agreement with the measured results. Using the practical observed data, the elastic modulus at the site can be obtained. The hardness of rock is determined depending on the value of the elastic modulus. The parameter identification presented in this paper would give one of the helpful tool for the construction of rock fill dam.

References


Fig.1: Topography of the Futatsuishi site
Fig. 3: Observed Points

Fig. 4: Prediction Model

Fig. 5: External Force
Fig. 6: History of Elastic Moduli

Fig. 7: History of Performance Function

Fig. 8: History of Velocity
Fig. 9: History of Elastic Moduli

Fig. 10: History of Performance Function
Table-1: Result of Identification

<table>
<thead>
<tr>
<th></th>
<th>CL</th>
<th>CM</th>
</tr>
</thead>
<tbody>
<tr>
<td>elastic wave [km/s]</td>
<td>0.6~1.0</td>
<td>1.0~2.1</td>
</tr>
<tr>
<td>elastic modulus [kN/m²]</td>
<td>~1.0 × 10⁷</td>
<td>1.0 × 10⁷~2.5 × 10⁷</td>
</tr>
<tr>
<td>result of identification [kN/m²]</td>
<td>3.9 × 10⁶</td>
<td>9.8 × 10⁶</td>
</tr>
</tbody>
</table>

Fig.11: History of Velocity