Optimal Control Applied to Water Flow Using Second Order Adjoint Equation

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Abstract

This paper presents the optimal control applied to the water flow using the adjoint Newton method. The method is based on the first and second order adjoint equations allowing to obtain a better approximation to the Newton line search direction. The gradient $\nabla J$ of the performance function $J$ is obtained by the first order adjoint equation. The Hessian/Vector Products is calculated by using the second order adjoint equation. The performance of the adjoint Newton method is compared with the Sakawa-Shindo method. The message passing interface library is used for the communication of parallel computing.

Key words : Adjoint Newton Method, First Order Adjoint Equation, Second Order Adjoint Equation, Tangent Linear Model Equation, Parallel Computing

1 Introduction

In many engineering problems, some observation points are set and the state values of there points are observed. Moreover, it’s often the case that initial and boundary conditions are unknown. At this time, the problems to find the state values so as to become the nearest the target values can be set. The differential of the values between the state values and the target values are calculated by the performance function. Usually, the performance function is expressed by the square residual between the target values and calculated the state values. The state that the performance function becomes the minimum is the target state.

In this paper, two optimal control problems are proposed. One is the optimal control of the water gate of the Agigawa dam. Another one is the water quality purification problem applied to Sasame river. The management of water in reservoir is very important for our life. The main purposes of the reservoir are hydroelectric power, water supply, irrigation, flood control and so on. It is called multi-purpose dam that is shared with these plural uses, and the maintenance of the dam is very complicated. However, if the storage of water at a reservoir is nearly full, a wrong operation may cause a dam breaking and loss of water. Therefore, the optimal control is required as being satisfied with each different demands. Though there are many control purposes of reservoirs, the control purpose is proposed for the problem of the reflection of waves in this paper. The water gate of the dam is opened when a flood comes in, and that is closed when a flow falls in. However, if the water gate of the dam is closed suddenly, the waves are reflected by the water gate of dam, and the water elevation is gone up in the reservoir. Therefore, the main purpose of the optimal control of the water gate of the dam is to reduce the water elevation in the reservoir and not to cause the reflection of waves.

Recently, the city’s rapid growth caused the water pollution makes progress by the domestic wastewater and the mud deposited at the bottom in the river. The stink and the scum occur because Dissolved Oxygen (DO) drops to a lower value. Moreover, the water volume is decreasing by the development of the sewer. In order to improve of the river environment, Dissolved Oxygen is increased by the discharge from the conducting tube. Therefore, the purpose is to increase Dissolved Oxygen in the river by the discharge from the conducting tube. However, it takes long computational time and larger
computer memory to carry out control problems. Therefore, the purpose is to reduce the computational time by the adjoint Newton method and the computer memory by the parallel computing.

The optimal control theory is used to obtain the control values. The adjoint Newton method is applied as a minimization technique. This method is suitable for the minimization problem of the performance function. The method is based on the first and second order adjoint techniques allowing to obtain a better approximation to the Newton line search direction. The gradient of the performance function is obtained by the first order adjoint equation. The Hessian/Vector Products is calculated by the second order adjoint equation. The adjoint Newton method is compared with the Sakawa-Shindo method.

## 2 State Equation

The non-linear shallow water equation and the advection diffusion equation are used for the analysis of flow behavior and material flow.

\[
\begin{align*}
\dot{q}_i - u_i u_j (\xi + H)_{,j} + u_i q_{j,j} + u_j q_{i,j} & - \nu \{ q_{i,jj} + q_{j,ij} - u_i (\xi + H)_{,jj} - u_j (\xi + H)_{,ij} \} \\
& + g(\xi + H)(\xi + H + \eta)_i + f q_i = 0 \\
\end{align*}
\]

(1)

\[
\begin{align*}
\dot{\xi} + q_{i,i} & = 0 \\
\end{align*}
\]

(2)

\[
\begin{align*}
\dot{c} + (u_c)_{,i} - \kappa c_{,ii} & = 0 \\
\end{align*}
\]

(3)

where, \( q_i, u_i, \xi, \eta, g, c \) and \( \kappa \) are the discharge of unit width, the water velocity, the water elevation, the bed elevation, the gravitational acceleration, Dissolved Oxygen and the diffusion coefficient, respectively. The coefficient of kinematic eddy viscosity \( \nu \) and \( f \) are expressed as follows:

\[
\begin{align*}
\nu & = \frac{k_l}{6} u_s (\xi + H), \\
f & = \frac{u_s}{(\xi + H)} \\
\end{align*}
\]

(4)

where, \( k_l \) and \( u_s \) are the Karman constant and the friction velocity, respectively. The friction velocity \( u_s \) is expressed as follows:

\[
\begin{align*}
u = \frac{n^2 \sqrt{u_k u_k}}{(\xi + H)^{1/3}}
\end{align*}
\]

(5)


## 3 Spatial Discretization

As for the spatial discretization, the bubble function element is applied for the discharge, the water elevation and Dissolved Oxygen. The bubble function interpolation is expressed as follows:

\[
q_i = \Phi_1 q_{i1} + \Phi_2 q_{i2} + \Phi_3 q_{i3} + \Phi_4 \tilde{q}_i
\]

(6)

\[
\tilde{q}_i = q_i4 - \frac{1}{3}(q_{i1} + q_{i2} + q_{i3})
\]

(7)

\[
\begin{align*}
\xi = \Phi_1 \xi_1 + \Phi_2 \xi_2 + \Phi_3 \xi_3 + \Phi_4 \tilde{\xi}_4 \\
\tilde{\xi}_4 = \xi - \frac{1}{3}(\xi_1 + \xi_2 + \xi_3)
\end{align*}
\]

(8)

(9)

\[
\begin{align*}
c = \Phi_1 c_1 + \Phi_2 c_2 + \Phi_3 c_3 + \Phi_4 \tilde{c}_4 \\
\tilde{c}_4 = c - \frac{1}{3}(c_1 + c_2 + c_3)
\end{align*}
\]

(10)

(11)

\[
\Phi_1 = L_1, \quad \Phi_2 = L_2, \quad \Phi_3 = L_3, \quad \Phi_4 = 27L_1L_2L_3
\]

(12)

where, \( \Phi_n(\alpha = 1, 2, 3, 4) \) is the bubble function for the discharge, the water elevation and Dissolved Oxygen, respectively.

## 4 Stabilized Bubble Function

The bubble function is capable of eliminating the barycenter point by using the static condensation. The discretized form derived by the bubble function is equivalent to those by SUPG method. The stabilized parameter which is derived by the bubble function is expressed as follows:

As for the momentum equation of the shallow water equation

\[
\tau_e B_{q_i} = \frac{\langle \dot{\phi}_e, 1 \rangle_{\Omega_e} A_e^{-1}}{\Delta t \| \phi_e \|_{L_2}^2 + \frac{1}{2} (\nu + \kappa) \| \phi_e \|_{L_2}^2 + f \| \phi_e \|_{L_2}^2} \\
\]

(13)

and continuity equation of the shallow water equation

\[
\tau_e B_{\xi} = \frac{\langle \dot{\phi}_e, 1 \rangle_{\Omega_e} A_e^{-1}}{\Delta t \| \phi_e \|_{L_2}^2 + \frac{1}{2} (\nu + \kappa) \| \phi_e \|_{L_2}^2} \\
\]

(14)

and advection diffusion equation

\[
\tau_e B_c = \frac{\langle \dot{\phi}_e, 1 \rangle_{\Omega_e} A_e^{-1}}{\Delta t \| \phi_e \|_{L_2}^2 + \frac{1}{2} (\nu + \kappa) \| \phi_e \|_{L_2}^2} \\
\]

(15)
where, $\hat{\nu}$ is the stabilized control parameter. From the stabilized parameter comparable to the SUPG method, an optimal parameter can be given as follows:

As for the momentum equation of the shallow water equation

$$\tau_e B_u = \left( \frac{1}{2} \tau^{\text{es}} - 1 + \frac{\alpha}{\Delta t} \right)^{-1}$$  \hspace{1cm} (16)

$$\tau^{\text{es}} = \left[ \left( \frac{2|U_i|}{h_e} \right)^2 + \left( \frac{4\nu}{h_e^2} \right)^2 + \left( \frac{u^*}{\xi + H} \right)^2 \right]^{\frac{1}{2}} \hspace{1cm} (17)$$

and continuity equation of the shallow water equation

$$\tau_e B_\xi = \left( \frac{1}{2} \tau^{\text{es}} - 1 + \frac{\alpha}{\Delta t} \right)^{-1}$$  \hspace{1cm} (18)

$$\tau^{\text{es}} = \left[ \left( \frac{2|U_i|}{h_e} \right)^2 + \left( \frac{4\nu}{h_e^2} \right)^2 + \left( \frac{u^*}{\xi + H} \right)^2 \right]^{\frac{1}{2}} \hspace{1cm} (19)$$

and advection diffusion equation

$$\tau_e B_c = \left( \frac{1}{2} \tau^{\text{exc}} - 1 + \frac{\alpha}{\Delta t} \right)^{-1}$$  \hspace{1cm} (20)

$$\tau^{\text{exc}} = \left[ \left( \frac{2|U_c|}{h_e} \right)^2 + \left( \frac{4\nu}{h_e^2} \right)^2 \right]^{\frac{1}{2}} \hspace{1cm} (21)$$

where,

$$\alpha = \frac{A_e}{\langle \phi_e, 1 \rangle_{\Omega_e}}$$  \hspace{1cm} (22)

$$h_e = \sqrt{2A_e}$$  \hspace{1cm} (23)

$$|U_i| = \sqrt{q_1^2 + q_2^2 + g(\xi + H)}$$  \hspace{1cm} (24)

$$|U_c| = \sqrt{u^2 + v^2}$$  \hspace{1cm} (25)

Therefore, many time steps can be taken in the computation.

6 Hessian/Vector Products

For easy of equation, the diffusion equation is applied to the formulation of the Hessian/Vector Products. The diffusion equation can be written as follows:

$$\dot{c} - \kappa c_{,tt} = 0 \hspace{1cm} (26)$$

$$c(t_0) = c^0 \hspace{1cm} (27)$$

$$c^*(t) = U_i \hspace{1cm} (28)$$

where, $c$ is the state value, $c^0$ is the initial condition, $U_i$ is the control value.

The performance function $J$ is defined as follows:

$$J = \frac{1}{2} \int_{t_0}^{t_f} \int_{\Omega} (c - c^{obj})^T W (c - c^{obj}) d\Omega dt \hspace{1cm} (29)$$

where, $c^{obj}$ is the target value at the object point and $W$ is the weighting diagonal matrix.

The purpose of this research is to find the optimal control value $U_i$ so as to minimize the performance function $J$ under the constraints of Eqs. (26)-(28).

The Lagrange multiplier method is suitable for the minimization problem of the performance function. The Lagrange multiplier is used to solve a minimization problem. Therefore, the performance function is extended by the Lagrange multiplier and the finite element equation. The extended performance function is expressed as follows:

$$J^* = J + \int_{t_0}^{t_f} \int_{\Omega} \lambda^T (\dot{c} - \kappa c_{,tt}) d\Omega dt \hspace{1cm} (30)$$

where, $c$, $\lambda$ and $J^*$ are considered as follows:

$$c = c + \delta c + \delta^2 c + ...$$

$$\lambda = \lambda + \delta \lambda + \delta^2 \lambda + ...$$

$$J^* = J^* + \delta J^* + \delta^2 J^* + ... \hspace{1cm} (31)$$

in this paper, the above equations are set as follows:

$$c = c^{(0)} + c^{(1)} + c^{(2)} + ...$$

$$\lambda = \lambda^{(0)} + \lambda^{(1)} + \lambda^{(2)} + ...$$

$$J^* = J^{(0)} + J^{(1)} + J^{(2)} + ...$$

$$= J^{(0)} + J^{(1)} + J^{(2)} + J^{(2)} + J^{(3)} + ...$$

$$= J^{**} \hspace{1cm} (32)$$

5 Temporal Discretization

To solve the state equation and the adjoint equation, the Crank-Nicolson method is used for the temporal discretization. This method is capable of taking the long time increment and superior in stability.
These equations are substituted for the extended performance function.

Therefore, \( J^{**} \) is expressed as follows:

\[
J^{**} = \frac{1}{2} \int_{t_0}^{t_f} \int_{\Omega} (c(0) + c(1) + c(2) - c^{obj})^T \nabla (c(0) + c(1) + c(2) - c^{obj}) d\Omega dt + \int_{t_0}^{t_f} \int_{\Omega} \lambda(0)^T \{ \hat{c}(0) + \hat{c}(1) + \hat{c}(2) \} d\Omega dt - \kappa \hat{c}_{,ii} + c_{,ii} + c_{,ii} \} d\Omega dt
\]

\[Eq.(33)\]

\( J^{(0)} \) is expressed as follows:

\[
J^{(0)} = \frac{1}{2} \int_{t_0}^{t_f} \int_{\Omega} (c(0) - c^{obj})^T \nabla (c(0) - c^{obj}) d\Omega dt + \int_{t_0}^{t_f} \int_{\Omega} \lambda(0)^T \{ \hat{c}(0) - \kappa \hat{c}_{,ii} \} d\Omega dt \quad \text{(34)}
\]

This equation is extended performance function.

### 6.1 First Order Adjoint Equation

\( J^{(1)} \) is expressed as follows:

\[
J^{(1)} = \int_{t_0}^{t_f} \int_{\Omega} \{-\dot{\lambda}(0)^T - \kappa \lambda_{,ii}^{(0)}\} + (c(0) - c^{obj})^T Q \ n_i c(1) d\Omega dt + \int_{t_0}^{t_f} \int_{\Omega} \lambda(1)^T (t_f) c(1) (t_f) - \lambda(0)^T (t_0) c(1) (t_0) \} d\Omega dt + \int_{t_0}^{t_f} \int_{\Omega} \lambda(0)^T (t_f) c(1) (t_f) - \lambda(0)^T (t_0) c(1) (t_0) \} d\Omega dt + \int_{t_0}^{t_f} \int_{S} \{ \kappa \lambda_{,i}^{(0)} n_i c(1) \} dS dt - \int_{t_0}^{t_f} \int_{S} \{ \kappa \lambda_{,i}^{(0)} c_{,i} (1) n_i \} dS dt + \int_{t_0}^{t_f} \int_{S_c} \{ \kappa \lambda_{,i}^{(0)} n_i U(1) \} dS_c dt \quad \text{(35)}
\]

Considering each term equal to zero, stationary conditions are obtained as follows:

\[-\dot{\lambda}(0)^T - \kappa \lambda_{,ii}^{(0)} + (c(0) - c^{obj})^T Q = 0 \text{ in } \Omega \quad \text{(36)}
\]

\[\dot{c}(0) - \kappa c_{,ii}^{(0)} = 0 \text{ in } \Omega \quad \text{(37)}
\]

\[\lambda(0)^T (t_f) = 0 \text{ in } \Omega \quad \text{(38)}
\]

\[\lambda(0)^T (t) = 0 \text{ on } S \quad \text{(39)}
\]

\[\kappa \lambda_{,i}^{(0)} n_i = 0 \text{ on } S \quad \text{(40)}
\]

Eq.(36) is the first order adjoint equation. \( J^{(1)} \) is expressed as follows by Eqs.(36)-(40).

\[
J^{(1)} = \int_{t_0}^{t_f} \int_{S_c} \{ \kappa \lambda_{,i}^{(0)} n_i U^{(1)} \} dS_c dt \quad \text{(41)}
\]

Therefore, the gradient is obtained by using the following equation.

\[\text{grad}(J) = \kappa \lambda_{,i}^{(0)} n_i \text{ on } S_c \quad \text{(42)}\]

### 6.2 Second Order Adjoint Equation

\( J^{(2)}_1 \) is expressed as follows:

\[
J^{(2)}_1 = \int_{t_0}^{t_f} \int_{\Omega} \{-\dot{\lambda}(0)^T - \kappa \lambda_{,ii}^{(0)}\} + (c(0) - c^{obj})^T Q \ c(2) d\Omega dt + \int_{t_0}^{t_f} \int_{\Omega} \lambda(2)^T (\dot{c}(0) - \kappa c_{,ii}^{(0)}) d\Omega dt + \int_{\Omega} \{ \lambda(0)^T (t_f) c(2) (t_f) - \lambda(0)^T (t_0) c(2) (t_0) \} d\Omega dt + \int_{t_0}^{t_f} \int_{S} \{ \kappa \lambda_{,i}^{(0)} n_i c(2) \} dS dt - \int_{t_0}^{t_f} \int_{S} \{ \kappa \lambda_{,i}^{(0)} c_{,i} (2) n_i \} dS dt + \int_{t_0}^{t_f} \int_{S_c} \{ \kappa \lambda_{,i}^{(0)} n_i U(2) \} dS_c dt \quad \text{(43)}
\]

\( J^{(2)}_1 \) is expressed as follows by Eqs.(36)-(40).

\[
J^{(2)}_1 = \int_{t_0}^{t_f} \int_{S_c} \{ \kappa \lambda_{,i}^{(0)} n_i U(2) \} dS_c dt \quad \text{(44)}
\]
Considering each term equal to zero, stationary conditions are obtained as follows:

\[ J^{(2)}_2 = \int_{t_0}^{t_f} \int_{\Omega} \left( -\dot{\lambda}^{(1)T} - \kappa \lambda^{(1)T}_{i,i} + c^{(1)^T}Q \right) c^{(2)}d\Omega dt \]
\[ + \int_{t_0}^{t_f} \int_{\Omega} \lambda^{(2)T} \left( c^{(1)} - \kappa c^{(1)}_{i,i} \right) d\Omega dt \]
\[ + \int_{t_0}^{t_f} \int_{\Omega} \left[ \lambda^{(1)T} (t_f) c^{(2)}(t_f) - \lambda^{(1)T} (t_0) c^{(2)}(t_0) \right] \right] d\Omega \]
\[ + \int_{t_0}^{t_f} \int_{S} \left[ (\kappa \lambda^{(1)T}_{i,i} n_i c^{(2)}(t_0)) \right] dSdt \]
\[ - \int_{t_0}^{t_f} \int_{S} \left[ (\kappa \lambda^{(1)T}_{i,i} n_i c^{(2)}(t_0)) \right] dSdt \]
\[ + \int_{t_0}^{t_f} \int_{S_c} \left[ (\kappa \lambda^{(1)T}_{i,i} n_i U^{(2)}(t)) \right] dS_c dt \] (45)

\[ J^{(2)}_3 \] is expressed as follows by the tangent linear model equation.

\[ J^{(2)}_3 = \frac{1}{2} \int_{t_0}^{t_f} \int_{\Omega} c^{(1)^T}Qc^{(1)}d\Omega dt \] (54)

### 7 Minimization Technique

The adjoint Newton method and Sakawa-Shindo method are used as the minimization technique. Algorithms are expressed as follows:

#### 7.1 Algorithm of Adjoint Newton Method

In the Newton method, the Newton line search direction \( \tilde{d}_k \) is calculated using the Newton equation.

\[ G_k \tilde{d}_k = -\tilde{g}_k, \] (55)

where, \( G_k \) is the Hessian matrix of the performance function, \( \tilde{d}_k \) is the Newton line search direction and \( \tilde{g}_k \) is the gradient of the performance function.

The main steps of the adjoint Newton algorithm is as follows:

Step 1. Chose the initial control value \( \tilde{U}_0 \), and set the iteration counter to \( k = 0 \).
Step 2. Test \( \tilde{U}_k \) for convergence. If the following convergence criterion is satisfied

\[ ||\tilde{U}_k - \tilde{U}_{k-1}|| < 10^{-6} \] (56)

then stop. Otherwise continue.
Step 3. Solve approximately the Newton Eq.(55) using Lanczos Method where the Hessian/Vector Products is obtained using the Second Order Adjoint Method.

Step 4. Set \( k = k + 1 \) and update

\[ \tilde{U}_{k+1} = \tilde{U}_k + \alpha_k \tilde{d}_k, \] (57)

Go to step 2.

#### 7.2 Algorithm of Sakawa-Shindo Method

The algorithm of Sakawa-Shindo method is shown as follows:

\[ J^{(2)}_2 \] is expressed as follows:

\[ \lambda^{(1)T} - \kappa \lambda^{(1)T}_{i,i} + c^{(1)^T}Q = 0 \text{ in } \Omega \] (46)
\[ c^{(1)} - \kappa c^{(1)}_{i,i} = 0 \text{ in } \Omega \] (47)
\[ \lambda^{(1)T} (t_f) = 0 \text{ in } \Omega \] (48)
\[ \lambda^{(1)T} (t) = 0 \text{ on } S_1 \] (49)
\[ \kappa \lambda^{(1)T}_{i,i} n_i = 0 \text{ on } S_2 \] (50)

Eq.(47) is the tangent linear model equation. Eq.(46) is the second order adjoint equation. \( J^{(2)}_2 \) is expressed as follows by Eqs.(46)-(50).

\[ J^{(2)}_2 = \int_{t_0}^{t_f} \int_{S_c} (\kappa \lambda^{(1)T}_{i,i} n_i U^{(2)}(t)) dS_c dt \] (51)

Therefore, the Hessian/Vector Products is obtained by using the following equation.

\[ \text{Hessian/Vector Products} (J) = \kappa \lambda^{(1)T}_{i,i} n_i \text{ on } S_c \] (52)

\[ J^{(2)}_3 \] is expressed as follows:

\[ J^{(2)}_3 = \frac{1}{2} \int_{t_0}^{t_f} \int_{\Omega} c^{(1)^T}Qc^{(1)}d\Omega dt \]
\[ + \int_{t_0}^{t_f} \int_{\Omega} \lambda^{(1)T} (\dot{c}^{(1)} - \kappa c^{(1)}_{i,i}) d\Omega dt \] (53)
Step 1. Chose the initial control value \( \vec{U}_0 \), and set the iteration counter to \( k = 0 \).
Step 2. Solve the state values by using the state equation.
Step 3. Compute the initial performance function.
Step 4. Solve the Lagrange Parameter by using the adjoint equation.
Step 5. Compute the identified control value \( \vec{U}_{i} \).
Step 6. Test \( \vec{U}_{i} \) for convergence. If the following convergence criterion is satisfied
\[
||\vec{U}_{i} - \vec{U}_{i-1}|| < 10^{-6} \tag{58}
\]
then stop. Otherwise continue.
Step 7. Solve the state values by using the state equation.
Step 8. Compute the performance function.
Step 9. Renew a weighting parameter \( W \).

### 8 Lanczos Method

The Newton equation can be written as follows:
\[
G_k \vec{U}_k = - \vec{g}_k, \tag{59}
\]
In this paper, \( \vec{U}_k \) is the same value at all control points. Therefore, \( \vec{d}_k \) can be expressed as follows:
\[
\vec{d}_k = \kappa \vec{U}_k^{T}, \tag{60}
\]
where, \( \vec{U}^{T} \) is the perturbation. Eq.(60) is substituted for Eq.(59). Therefore, Eq.(59) becomes as follows:
\[
G_k \kappa \vec{U}_k^{T} = - \vec{g}_k, \tag{61}
\]
where, \( G_k \vec{U}_k^{T} \) is the Hessian/Vector Products. And, multiplying both sides by \( \vec{U}_k^{T} \) gives the following equation.
\[
\vec{U}_k^{T} \kappa \vec{h}_k = - \vec{U}_k^{T} \vec{g}_k, \tag{62}
\]
where, \( \vec{h}_k \) is the Hessian/Vector Products. \( \kappa \) is obtained by the following equation.
\[
\kappa = - \frac{\vec{U}_k^{T} \vec{g}_k}{\vec{U}_k^{T} \vec{h}_k}, \tag{63}
\]
The line search direction is obtained by the above equation. Therefore,
\[
\vec{d}_k = \kappa \vec{U}_k^{T}, \tag{64}
\]
\[
= \frac{\vec{U}_k^{T} \vec{g}_k}{\vec{U}_k^{T} \vec{h}_k} \vec{U}_k^{T}. \tag{64}
\]

### 9 Parallel Computing

Parallel computer is that some computers using high performance network are bundled. Each computer has processor and computational memory respectively, and computes with communicating each processor. Therefore, it is possible to calculate a large scale and high performance computing. In this study, IBM series 690 is used for parallel computing and message passing interface( MPI ) is used for message passing library. MPI is a message passing library used for parallel processing distributed memory system. The parallel computing technique based on the domain decomposition technique is developed in order to reduce the computational time and computational storage requirement. Thus, it is important to minimize quantity of communication among processors and equalize the computational load in each processors.

#### 10.1 Numerical Example 1

Agigawa dam which locates in South East part of Gifu prefecture is used as a computational model. Agigawa dam is shown in Fig.1. Fig.2 shows the finite element mesh of the reservoir and the object points(pointA,B). The total number of nodes and elements are 2104 and 3609, respectively. The length of the reservoir is about 2 [km]. The whole control duration \([t_0, t_f]\) is assumed as 24[h]. The time increment \( \Delta t \) is 1.0 [sec] and the total number of time step is 86400 [step]. The weighting diagonal matrix \( W \) is used as 1.0I. Fig.3 shows the time history of the inflow discharge on the boundary \( S_{B1} \) and \( S_{B2} \). Controlling the outflow at the water gate, the water elevation can be reduced in the reservoir. The adjoint Newton method and the Sakawa-Shindo method are used as the minimization technique. The parallel computer is used. Total number of computers are 16 nodes. Fig.4 shows the comparison of the variation of two performance functions. Fig.5 shows the time history of the control discharge at the water gate of the dam on the boundary \( S_{C} \). Figs.6 and 7 show the time history of the water elevation at the object points(pointA,B).

#### 10.2 Numerical Example 2
Sasame river which locates in Saitama prefecture is used as a computational model. Sasame river is shown in Fig.8. Fig.9 shows the finite element mesh of Sasame river and the object point. The total number of nodes and elements are 3505 and 5600, respectively. The length of the river is about 3.6 [km]. The whole control duration \([t_0, t_f]\) is assumed as 24[h]. The time increment \(\Delta t\) is 1.0 [sec] and the total number of time step is 86400 [step]. As a initial condition, Dissolved Oxygen( DO ) constantly is 2.0(mg/l) at the whole domain. Dissolved Oxygen sets to 2.0(mg/l) and 6.0(mg/l) on the boundary \(S\) and \(S_c\). Controlling the discharge form the conducting tube, Dissolved Oxygen can be increased in the river. The adjoint Newton method and the Sakawa-Shindo method are used as the minimization technique. The parallel computer is used. Total number of computers are 16 nodes. Fig.10 shows the time history of the inflow discharge on the boundary \(S\). Fig.11 shows the comparison of the variation of two performance functions. Fig.12 shows the time history of the control discharge on the boundary \(S_c\). Fig.13 shows the time history of Dissolved Oxygen at the object point.

11 Conclusion

In this paper, the optimal control by using the second order adjoint equation is proposed. The adjoint Newton method is based on the first and the second order adjoint equations allowing to obtain a better approximation to the Newton line search direction. The performance of the adjoint Newton method is compared with the Sakawa-Shindo method. As the numerical example 1, the optimal control problem of the water gate of a dam is presented. The optimal control of water gate of Agigawa dam is successful. The computational time obtained by the adjoint Newton method and the Sakawa-Shindo method are about 57.6[h] and 111.2[h], respectively. As the numerical example 2, the water quality purification problem is presented. The optimal control of Dissolved Oxygen of Sasame river is successful. The computational time obtained by the adjoint Newton method and the Sakawa-Shindo method are about 85.3[h] and 175.8[h], respectively. In two problems, the computational time obtained by the adjoint Newton method was faster than that by the Sakawa-Shindo method.

References


Fig. 6 Water-Elevation at Point A

Fig. 7 Water-Elevation at Point B

Fig. 8 Sasame River

Fig. 9 Finite Element Mesh
Fig. 10 Inflow Discharge on S

Fig. 11 Performance Function $J$

Fig. 12 Control Discharge on $S_c$

Fig. 13 Dissolved Oxygen at Objective Point