Automatic Mesh Generation Using Delaunay Tetrahedrization

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Abstract

This paper presents an introduction of 3-dimensional Delaunay Tetrahedrization which is a technique of Mesh Generation for numerical analysis. The Finite Element Method is one of the analytical approach with finite element mesh. Generally, the Mesh Generation takes a lot of time and labor. This will be an important problem when the future. The Delaunay Tetrahedrization is a technique to divide the domain into the tetrahedron by using a set of the nodes that are distributed arbitrarily. This technique can be applied to generate the finite element mesh of complicated object, that is to say, the practical model. Purpose of this study is to introduce how to generate finite element mesh of practical models and to investigate the effectiveness of the generated finite element meshes.
1 Introduction

In recent years, the performance of electric computer is drastically improved and the technology of the numerical analysis is highly progressed. Therefore it is possible to carry out the large-scale numerical simulation. The Finite Element Method has been successfully applied to solve a variety of engineering problems. On the other hand, it is depend on the finite element mesh. In a day and age, analytical objects are complicated, so it is impossible to generate the finite element mesh by hand. Automatically mesh generation is necessary for Finite Element Method at this time. The Delaunay Triangulation is an effective technique of mesh generation. They are introduced in following section.

2 DELAUNAY DIVIDING

Delaunay subdividing is one of the technique to divide domains. Triangular elements are generated with the node group arbitrarily set in computational domains. Circumcircle of a triangle generated by Delaunay Triangulation does not include nodes of other elements. (Fig.1) This is a feature of Delaunay Triangulation. Fig.2 shows an example that is not Delaunay Triangulation. By using this method, the triangles are nearly an equilateral triangles can be obtained. In the 3-dimensional mesh generation, Delaunay Tetrahedrization is often used. Circumscribed sphere of a tetrahedron generated by Delaunay Tetrahedron does not include nodes of other elements. This geometry is agreement with preferable shape for Finite Element Method.

2.1 Creation of the Triangles by using Supertriangle

Supertriangle is effective in dividing computational domain into triangles. The Supertriangle takes in computational domain. (Fig.3) As the first step of Delaunay Triangulation, if one node is set, at least one triangle disappears, and at least three triangles are generated. (Fig.4) When Delaunay triangulation ends for all nodes, triangles included composition point of Supertriangle are removed.

![Fig.1 : Example of Delaunay Triangulation](image1)

![Fig.2 : Example that is not Delaunay Triangulation](image2)
2.2 Surface Generation

Delaunay Triangulation is done only to the nodes on the boundary, it is done without thought of
domain’s concavity and convexity. Another way of saying, Delaunay Triangulation has no concept
of boundary. So, definition of boundary must be set up. 2-dimensional domain has a number of
boundaries. Circumference of computational domain is defined as an exterior boundary. The other
boundaries are defined as interior boundaries. First step is to number the nodes on boundaries.
As for the exterior boundary, nodes are numbered clockwise. On the other hand, nodes on the
interior boundaries are numbered counterclockwise. The nodes that compose a triangle makes it
anti-clockwise connect in all triangles.

If a triangle is generated by nodes on
only exterior boundary, it can be judged
whether it is an inside or it is the outside
of the computational domain clockwise of
the order the node on the boundary or
anti-clockwise.(Fig.5) As for the interior
boundaries as well as the exterior, it can
be judged whether the triangle is outside
or inside of computational domain.

2.3 Swapping Algorithm

This is the technique of the replacement. The node P is set into the computational domain which is
divided by Delaunay Triangulation, a triangle is divided into three triangles centering the node P.
If the circumcircle of these triangles enclose the nodes that compose another triangle, replacement
must be done.

Process of the replacement is

[1]Search of the triangle which is enclosed in circumcircle of the triangles composed by the node
P.

[3] Creation of the new triangles by connecting the node P and apexes of polygon.
[4] Repeat [1], [2], and [3], for the new triangle.
[5] End of Delaunay Triangulation by setting the node P.

Fig.6 : Swapping Algorithm

3 REFINEMENT OF FINITE ELEMENT MESH

It is necessary to generate more fine mesh to improve accuracy of numerical analysis. Because manual labor takes a lot of time and energy, it is necessary to generated nodes automatically inside the computational domain. And, the finite element mesh should be nearly the equilateral triangle. So, distorted triangle should be adjusted. As a technique of generation the nodes in the computational domain, Radiation Datum Line Method (Junichi Matsumoto) is applied. And, the Laplacian Method is applied to adjust the nodes.

3.1 Radiation Datum Line Method

The Radiation Datum Line Method is one of the generation method in 2-dimensional form. At first, Base line is installed. When quarter sector include computational domain, distance and angle is used. (Fig.7) Nodes are placed evenly spaced apart in a radial pattern.

Fig.7 : Radiation Datum Line Method
3.2 Laplacian Method

Fine mesh generated by Delaunay Triangulation and Radiation Datum Line Method usually has distortion. The triangle which has distortion should be adjusted. One Node constructs some triangles. It should be a center of these triangles which are constructed by the one. As for each triangle the node is moved so that a following equation is implemented. It is continued until calculation convergence.

$$P(i) = \frac{1}{2n} \sum_{j=1}^{n} \{P(j) + P(k)\}$$ (1)

![Fig.8 : Image of Laplacian Method](image)

4 MOUNTAIN AND TUNNEL MODEL

In this section, the process of generating mountain and tunnel model is introduced. There are basic five processes. The five processes are following, As a practical model, Suemune tunnel is taken an example. Suemune tunnel is located at Okayama, Japan. The surface data of Suemune tunnel is provided by Sato Co., Ltd. The provided data are altitude in the analytical domain and shape of tunnel. Fig.9 shows nodal distribution of surface.

1. Generate Surface nodal distribution.
2. Generate the nodal distribution. of tunnel.
3. Subdivide domain near the mine mouth.
4. Generate virtual nodes.
5. Generate 3dimensional mesh.

4.1 Nodal distribution of tunnel

From the provided data, nodal distribution of tunnel is generated. The shapes of tunnels are often rounded. To express roundness, the distance of each node are becoming short. Surface of mountain and the mine mouth of tunnel are near. In these area, each nodes’distance is incompatible. From fine nodal distribution and rough distribution, the qualities of elements generated is low. So, it is better to subdivide domain near the mine mouth.
4.2 Virtual nodes

The Delaunay tetrahedrization can't recognize the concave face. Therefore inadequate elements are generated on surface. To clear up these problem, virtual nodes are applied. Setting virtual nodes follow the concave face, inadequate elements are not generated on surface. After the mesh generation, elements that consist of virtual nodes are removed. Of course, virtual nodes are removed. Fig.11 shows inadequate elements on surface. These elements are on the virtual nodes. Fig.12 shows nodal distribution of surface and virtual nodes.
The virtual nodes are effective for not only surface but also interior boundaries. The virtual nodes are applied to the tunnel. Fig.13 shows tunnel model and virtual nodes. After mesh generation, the element which is generated in interior area are removed.

![Finite element mesh](image1)

Fig.14 : Finite element mesh

## 5 NUMERICAL EXAMPLE

It is necessary to verify the mesh generated by Delaunay Triangulation whether it is applicability to numerical analysis. Fig.15 shows Sendai river in Kagoshima, Japan. Finite element mesh(Fig.16) is modeled this river. Total number of nodes and elements are 1351 and 2371. Fig.17 shows closeup of an interior boundary.

![Sendai river](image2)

Fig.15 : Sendai river

![Finite Element Mesh](image3)

Fig.16 : Finite Element Mesh

![Closeup of Interior Boundary](image4)

Fig.17 : Closeup of Interior Boundary

The shallow water equation is applied to this model. Constant quantity of flow is given on S1.(Fig.16) Velocity vector is shown.(Fig.18 and 19) It can be said that the numerical analysis...
succeeded.

Fig.18 : Velocity Vector  
Fig.19 : Closeup of Interior Boundary

6 CONCLUSION

Finite element mesh is generated by Delaunay Triangulation and Tetrahedrization. Using Radiation Datum Line Method, the mesh is become fine. Mountain and tunnel model are generated by the Delaunay tetrahedrization. The way presented in this paper is effective for the mountain and tunnel model. This way can be applied to another practical model not only mountain and tunnel. As a numerical example, it is clear that the mesh can be used for numerical analysis. In this case, there are not only exterior boundaries but also interior boundaries.

References