An Analysis of FSI Problem Using ALE Finite-Element Method and Mesh Control Theory

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SUMMARY

This research presents an analysis of the circular cylinder located in the fluid flow as the Fluid-Structure interaction problem. The most important problem is the boundary reproduction method between fluid and structure for this problem. Arbitrary Lagrangian Eulerian (ALE) finite element Method is used to analyze the problem.

The research on an aerodynamic characteristic of the structure has been done by the experimental research, but in recent years, various characteristics are investigated by the numerical analyses. Although in the numerical analysis, various approaches has been developed, the definite conclusion has not been put out. It is necessary to consider the boundary reproduction method between fluid and structure for expressing the characteristic of the structure.

In this research, the ALE finite element method is applied as the boundary reproduction method. The ALE method makes up for the fault of the Lagrangian and Eulerian coordinate systems, and is a useful method for the moving boundary reproduction. In the ALE method, the movement of a continuous body is described with a reference coordinate system that can be arbitrarily moved without any relation to transformation of structure and movement of fluid. As a result, effectively both parties parts are combined and the description located in the flow of the transformation and flow becomes possible. Here, new mesh control approach is tried in this research to be smaller mesh strain. Development of this method is the main point of this research.

1 Introduction

The Fluid-Structure interaction problem is one of the moving boundary problem, and an important problem in engineering to know the behavior of boundary that changes depending on time. A research on the characteristic of the structure is a moving boundary problem, which is investigated by the experimental research. In recent years, various characteristics are cleaved by numerical analyses. Especially, a lot of problems are about the whirlpool excitation phenomenon of the structure for the design of the structure.

In this research, 2-dimensional circular cylinder in the unsteady flow is analyzed. The circular cylinder is assumed as solid body to which is elastically supported. Motion of the cylinder is described by the simple harmonic oscillation. Incompressible viscous flow of non-linear equation is obtained by the non-dimensional Navier-Stokes equations which is written by the ALE description. For the temporal discretization of the velocity of this momentum equation, the Crank-Nicolson method is applied to as implicit scheme, and the pressure and continuity equation explicit method is applied. The stabilized bubble function is applied to this equation to stabilize the computation. Mesh control is one of the most important and difficult problem of ALE method. Traditional mesh control methods are called down because of mesh strain by large displacement. Referring to these facts, ALE method is unfitted Large displacement problem. Here, in this research To be small mesh strain, Share Slip Mesh Update method (SSMU method) is applied to the rotation movement. A vertical movement mesh control method, Flexible Update and Re-Use Mesh Information method (FURUMI method) is developed by this research. By these two mesh control method, re-mesh can be completely carried out.
2 Governing Equation described by ALE

2.1 Navier-Stokes Equation

The incompressible Navier-Stokes equation is employed as a state equation. Let $\Gamma$ denote the boundary of $\Omega$, and suppose that an incompressible viscous flow occupied $\Omega$. The state equation of the flow can be written by the following Navier-Stokes equation in the non-dimensional form:

$$
\frac{\partial \mathbf{u}}{\partial t} + c \nabla \mathbf{u} + \nabla p - \nu \nabla \cdot (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) = \mathbf{f}, \quad \text{in } \Omega,
$$

where $\mathbf{u}$, $p$, and $\nu$ are the velocity, pressure, and the viscosity coefficient respectively, in which $\nu$ is the inverse of Reynolds number. $c$ is arbitrary material velocity on ALE coordinate system and shown as the ALE advection term equation:

$$
c \simeq \mathbf{v} - \hat{\mathbf{v}},
$$

here, $\hat{\mathbf{v}}$ is mesh velocity. Because in hte ALE method node coordinate can be selected arbitrary, $\hat{\mathbf{v}}$, is expressed by time derivative of coordinate $\mathbf{X}$ as

$$
\hat{v}_i = \frac{\partial X}{\partial t},
$$

where $\mathbf{X}$ is mesh coordinate.

2.2 Boundary Condition

This problem to be solved is represented in Fig.1. A solid body $B$ with the boundary $\Gamma_B$ is laid in an external flow, where $\hat{\mathbf{u}}$ is constant inflow velocity. Suppose that the boundary condition for this problem is given as:

$$
\mathbf{u} = (\hat{\mathbf{u}},0) \quad \text{on } \Gamma_1,
$$

$$
\mathbf{u}_2 = 0, \quad t_1 = 0 \quad \text{on } \Gamma_2,
$$

$$
t = 0 \quad \text{on } \Gamma_3,
$$

$$
\mathbf{u} = \mathbf{u}_B, \mathbf{v} = \mathbf{v}_B \quad \text{on } \Gamma_4,
$$

$$
t = \{ -p \mathbf{I} + \nu (\nabla \mathbf{u} + \nabla \mathbf{u}) \} \cdot \mathbf{n},
$$

where $\mathbf{t}$ is traction and $\mathbf{n}$ is unit vector of outward normal to $\Gamma$ respectively. The fluid forces subjected to the body are denoted by $\mathbf{F}$, where two components are drag and lift forces, respectively. The fluid force $\mathbf{F}$ is obtained by integrating the traction $\mathbf{t}$ on the boundary $\Gamma_B$:

$$
\mathbf{F} = - \int_{\Omega_B} \mathbf{t} d\Omega.
$$
2.3 Discretization

The discretization in space is carried out applying the Galerkin method to the momentum equation of the Navier Stokes equation. The finite element equation is expressed as:

\[ M \ddot{u}_i + (S + D) u_i - A p_i = f_i, \]  

(11)

where, \( M \) is mass, \( D \) is diffusion times \( \nu \), \( S \) is advection times material arbitrary velocity \( c \) and \( A \) is pressure, in which, \( \dot{u}_i \) and \( u_i \) is defined by,

\[ \dot{u}_i = \{ \dot{u}_i, \dot{v}_i \}^T, \quad u_i = \{ u_i, v_i, p_i \}^T. \]  

(12)

Thus, the Navier Stokes equation discretized by Galerkin method is expressed,

\[ M \ddot{u}_i + C u_i = f_i, \]  

(13)

\[ \dot{u}_i, u_i \text{ and } f_i \text{ is divided corresponding velocity nodes of solid body } \gamma \text{ (nodes on } \Gamma_B) \text{ and the other nodes } \alpha, \]

\[ \dot{u}_i = \{ \dot{u}_i^\alpha, \dot{u}_i^\gamma \}^T, \quad u_i = \{ u_i^\alpha, u_i^\gamma \}^T, \quad f_i = \{ f_i^\alpha, f_i^\gamma \}^T. \]  

(14)

From these division, Navier-Stokes equation is shown,

\[ \begin{bmatrix} M^{\alpha\alpha} & M^{\alpha\gamma} \\ M^{\gamma\alpha} & M^{\gamma\gamma} \end{bmatrix} \begin{bmatrix} \ddot{u}_i^\alpha \\ \ddot{u}_i^\gamma \end{bmatrix} + \begin{bmatrix} C^{\alpha\alpha} & C^{\alpha\gamma} \\ C^{\gamma\alpha} & C^{\gamma\gamma} \end{bmatrix} \begin{bmatrix} u_i^\alpha \\ u_i^\gamma \end{bmatrix} = \begin{bmatrix} f_i^\alpha \\ f_i^\gamma \end{bmatrix}. \]  

(15)

The Crank-Nicolson method is applied as the discretization in time to velocity of momentum equation. For continuity equation and pressure of momentum equation, explicit scheme is applied as temporal discretization.

2.4 Body Oscillation

A circular cylinder is solid body and assumed to have three-degree-of-freedom, which are displacement of \( x \) and \( y \), and rotation displacement \( \theta \), respectively.

The motion equation is shown;

\[ m \ddot{X} + c \dot{X} + kX = F, \]  

(16)

where, \( \dot{X} = \{ \dot{x}, \dot{y}, \dot{\theta} \}^T \), \( \ddot{X} = \{ \ddot{x}, \ddot{y}, \ddot{\theta} \}^T \), \( X = \{ x, y, \theta \}^T \), \( F = \{ F_x, F_y, M \}^T \).

And \( m \) is mass, \( c \) is damping, \( k \) is rigidity, respectively are diagonal matrix. \( F \) is external force.
And the relations of variables of solid body and fluid flow are following equations. Compatibility condition is expressed by

\[
\begin{align*}
\dot{u}_i^\gamma &= T^T \dot{X} = T^T \dot{u}, \\
u_i^\gamma &= T^T \ddot{X} = T^T u.
\end{align*}
\] (17)

Balance condition is expressed by

\[F + T f_i^\gamma = 0.\] (18)

where, \(T\) expresses the which is shown geometric relation between gravity center of the body and each node of material surface . Thus, \(F\) is expressed by;

\[
F = \left\{ F_x = \sum_{i=1}^{n} f_{x_i}, \quad F_y = \sum_{i=1}^{n} f_{y_i}, \quad M = \sum_{i=1}^{n} (y_i \cdot f_{x_i} - x_i \cdot f_{y_i}) \right\}^T
\] (19)

From equations (16), (17) and (18), motion of solid body is expressed by

\[m \dot{u} + c u + k X = -T f_i^\gamma,\] (20)

and from equation (19), equation (15) is expressed,

\[
\begin{bmatrix}
M^{\alpha\alpha} & M^{\alpha\gamma} \\
M^{\gamma\alpha} & M^{\gamma\gamma}
\end{bmatrix}
\begin{bmatrix}
\dot{u}_i^\alpha \\
T^T \dot{u}
\end{bmatrix}
+
\begin{bmatrix}
C^{\alpha\alpha} & C^{\alpha\gamma} \\
C^{\gamma\alpha} & C^{\gamma\gamma}
\end{bmatrix}
\begin{bmatrix}
u_i^\alpha \\
T^T u
\end{bmatrix}
=
\begin{bmatrix}
f_i^\alpha \\
f_i^\gamma
\end{bmatrix}.
\] (21)

Introducing, equation (21) to equation (20), it is obtained that

\[m^* \dot{u} + c^* u + k X = -T (M^{\gamma\alpha} \dot{u}_i^\alpha + M^{\gamma\gamma} T^T \dot{u} + C^{\gamma\alpha} u_i^\alpha + C^{\gamma\gamma} T^T u),\] (22)

Here,

\[
\begin{align*}
m^* &= m + T M^{\gamma\gamma} T^T, \\
c^* &= c + T C^{\gamma\gamma} T^T.
\end{align*}
\] (23)

\(m^*\) and \(c^*\) are the equivalent mass and damping which the influence of additional mass and damping by interaction effect between body and surroundings fluid. Rearranging these equations, the following equations are obtained.

\[M^{\alpha\alpha} \dot{u}_i^\alpha + M^{\alpha\gamma} T^T \dot{u} + C^{\alpha\alpha} u_i^\alpha + C^{\alpha\gamma} T^T u = f_i^\alpha,\] (24)

\[m^* \dot{u} + c^* u + k X = -T (M^{\alpha\alpha} \dot{u}_i^\alpha + C^{\alpha\gamma} u_i^\alpha).\] (25)

Solving the these equations fluid structure interaction problems can be obtained.

\[
\begin{bmatrix}
M^{\alpha\alpha} & M^{\alpha\gamma} T^T \\
T M^{\gamma\alpha} & m^*
\end{bmatrix}
\begin{bmatrix}
\dot{u}_i^\alpha \\
\dot{u}
\end{bmatrix}
+
\begin{bmatrix}
C^{\alpha\alpha} & C^{\alpha\gamma} T^T \\
T C^{\gamma\alpha} & c^*
\end{bmatrix}
\begin{bmatrix}
u_i^\alpha \\
u
\end{bmatrix}
=
\begin{bmatrix}
f_i^\alpha \\
f_i^\gamma
\end{bmatrix}
+k \begin{bmatrix}
X
\end{bmatrix}.
\] (26)
3 Mesh Control

3.1 Shear-Slip Mesh Update method

The Shear-Slip Mesh Update Method (SSMU method), designed to handle certain classes of flow problems with moving boundaries is applied ALE method. Specifically, this method is focused on problems with large and regular boundary displacement such as rotation. These motions are accommodated by using a thin layer of deforming space-time element, together with limited re-meshing without any projection at space-time slab interfaces.

The SSMU method is mesh designs which combine regions of rigid non-deforming elements with layers of shear-absorbing deforming elements. This accommodates the motions consisting of rotation. Similarly, a rotating object is embedded in a disk of rigid elements which rotate 'glued' to that object. These non-deforming regions are immersed in another set of non-deforming elements spanning the exterior boundaries, as shown in Fig.4. The thickness of this layer can span one or more elements.

A triangle-element shear-slip layer of located around circle is illustrated in Fig.5.

3.2 Flexible Update and Re-Use Mesh Information method

Flexible Update and Re-Use Mesh Information method (FURUMI method), applied SSMU method for straight movement of limited space is carried out for vertical transformation. In this method, mesh strain is minimized in space limited to some degree against one direction movement. And re-mesh cost and node movement is minimum too.
So large transformation moving boundary problem can be analyzed by this method only one direction movement. Application to two difference direction of this method and SSMU method, three degree of freedom moving boundary problem is solved in two dimension. Fig.6 shows fixed mesh, moving mesh and Flexible Update and Re-Use layer. Fixed mesh has some space for movement of moving mesh. When moving mesh is close to FURU layer node, FURU layer is reconstructed against as Fig.7 shows.

3.3 Laplace Equation

The mesh subdivision is candied on the basis of the potential. The boundary of fluid is set to 0 potential.

\[
\phi = 0 \text{ on } \Gamma_1, \quad \phi = 0 \text{ on } \Gamma_2, \quad \phi = 0 \text{ on } \Gamma_3, \quad \phi = \delta \text{ on } \Gamma_B, \quad (27)
\]

displacement of nodes in area obtain by the Laplace equation,

\[
\nabla \cdot \phi = 0, \quad (28)
\]

where, \(\Gamma_1, \Gamma_2, \Gamma_3\) are shown fixed boundary. \(\Gamma_B\) is shown moving boundary, and \(\delta\) is displacement of moving structure.

4 Numerical Study

4.1 Test Case for SSMU method

In this study, the behavior of the H type object is analyzed by the arbitrary Lagrangian Eulerian finite-element method using SSMU method. Because of the drag and lift forces, the H type object is rotated by centering on it in incompressible viscous flow. The computational domain and the finite element mesh are shown in Figs.8 and 9, where the mesh consists of 1882 nodes and 3594 element. In these Figures, computational conditions are shown. Reynolds number is 250.

4.2 Test Case Results

Test case is set to following parameters; H type object mass as object is 0.1, elastic of moment is 0.01, viscous damping coefficient is 0.0001, \(\Delta t\) is 0.01, all values are non-dimensional amount. Computing time is 0 to 30 as non-dimensional time. Test case results are shown in Figs 10-17. Fig 10 shows the velocity vector distribution at 30, and fig 11 represents the pressure distribution at 30. Figs 12 and 13 shows initial mesh and final mesh of around object.

Figs 14 is shown pressure distribution around object. Figs 15 is shown velocity distribution around object. Figs 16 is shown object rotated variation. Figs 17 is shown object angle against horizontal angle.
4.3 Numerical Analysis

In this study, the behavior of the circular cylinder is analyzed by the arbitrary Lagrangian Eulerian finite-element method using SSMU and FURUMI method. Because of the drag and lift forces, the circular cylinder is rotated by centering on it and vibrated up and down in incompressible viscous flow. The computational domain and the finite element mesh are shown in Figs.18 and 19, where the mesh consists of 2314 nodes and 443 element. In these Figures, computational conditions are shown. Reynolds number is 250.

4.4 Conclusion

Arbitrary Lagrangian Eulerian Finite-Element method can be applied to the analysis of the H type object rotation. The H type object rotated by momentum force from fluid, and would be stabilized angle step by step. Using SSMU method can be analyzed the large rotation problem at less mesh distortion. The cycle of rotated vibration is obtained from this study.
References


