Estimation of River Current
Using Reduced Kalman Filter Finite Element Method

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Abstract
The purpose of this paper is to investigate estimation of velocity, water elevation and contaminant concentration in a river current using the reduced Kalman filter finite element method, which is an improvement of the Kalman filter finite element method previously presented by the authors group. The state variables, velocity, water elevation and contaminant concentration, in the whole domain can be obtained by the finite element method. Combining the Kalman filter and the finite element method, a practically useful method can be obtained. The Kalman filter is an estimation method based on the system and observation equations. The non-linear shallow water equation is utilized for the state equation. The explicit Euler method is used for the temporal discretization. The finite element method based on the linear interpolation is employed for the spatial discretization. The long computational time is required for the computation by the previous method. To reduce the computational time, the computational domain is divided into two parts, the main and subsidiary domains. In the main domain, filtering procedures are carried out, whereas only a deterministic process is taken for the variables in the subsidiary domain. Eliminating the state variables in the subsidiary domain, the drastically efficient computation is carried out. The flow in the Teganuma river in Japan is computed as a practical application. Close agreement between observed and computed results was obtained.

Key Words: Kalman Filter, Finite Element Method, Reduce Method
Contaminant Concentration, Linear Shallow Water Equation, Teganuma River

1 INTRODUCTION
Recently, there are a lot of social demands for assessment on the occasion of new constructions especially in water areas, such as river, lake, shallow sea, etc. To satisfy such demands, numerical methods are one of the promising techniques. Careful observations at monitoring points in the area are also required. However, sometimes the observation is not satisfactory to know the physical phenomena in the whole area because the number of observation points is limited and observation errors are included in the data. To complement lack of the number of observations, numerical computations are one of the powerful tools for practical works.

Numerical results computed sometimes do not agree with measurements. There are sometimes unsolvable discussions between computationalists and observationalists whether the data measured are reliable or unrealistic to explain the physical phenomena. The observations include an amount of observation errors. Thus, the computation considering the system and observation errors is needed for the practical use. To exclude those errors from the computation considering the measurement data, the Kalman filter is one of the well established techniques.
Kalman (1960) presented the celebrated filtering procedure which is referred to as the Kalman filter and proved that the linear filter is optimal even if it is applied to the nonlinear system. This idea is very useful and frequently used in the field of meteorology and oceanology (e.g., Bengtson et. al. (1981), Ghil and Malanotte-Rizzoli (1991), Robinson and Lermusiaux (2000), Kao et. al. (2003), Kalnay (2003)). Because a lot of observation data are collected in those fields, the ensemble method is the present target of the research, e.g., Zupansky et. at. (2005). In the research of the prediction of contamination of the shallow water, although there are a large number of the areas to be considered, the observation points and data at each area are limited in usual. There are a few papers published including flow models in the Kalman filter. To compensate lack of the observation and to exclude the observation and system errors, the Kalman filter finite element method has been originated and applied to the practical problems (Hayakawa · Kawahara (1996), Takagi et. al. (1997), Nishiwaki · Kawahara (1997), Suma · Kawahara (1999), Fujimoto · Kawahara (2001), Yonekawa · Kawahara (2001), Suga · Kawahara (2003), Wakita · Kawahara (2004), Kato · Kawahara (2005)). The Kalman filter is a useful tool to estimate sequential average values and corresponding covariances. The advantage of a combination between the Kalman filter and the finite element method is useful for the computation in both time and spatial directions.

To apply the Kalman filter finite element method to practical use, the most annoying problem is to take long computational time especially for the matrix multiplication. To shorten the computational time, the domain decomposition is discussed (Fujimoto · Kawahara (2001)). To reduce the computational time based on the characteristic of the filter, the reduced Kalman filter finite element method is presented in this paper. The whole computational domain can be classified into two domains, i.e., one is the main domain, in which the computation is mainly carried out and the other is the subsidiary domain, in which the auxiliary computation by the finite element method is performed. The state variables at the nodal points in the subsidiary domain are assumed un-correlated with those in the main domain. Those variables in the subsidiary domain is used only for the computation by the finite element method. The state variables in the subsidiary domain are used for the deterministic process. The Kalman filter including deterministic variables can easily be derived. The idea is applied to the estimation of velocity, water elevation and contaminant concentration in the Teganuma river located in Japan. A flow of river can be modeled by the nonlinear shallow water equations. For the index of contaminant concentration, the dissolved oxygen, DO, is used because this index is very much related to habitation of living things in the water. To express the concentration of DO, the diffusion dispersion equation is employed. There are four observation points in the computed area of the Teganuma river, and at each point two horizontal components of velocity, water elevation and contaminant concentration are measured by the Ministry of Land, Infrastructure and Transport. Therefore, a total of 16 components of time series data are obtained. Based on the measurement data at 3 points, i.e., 12 components, the computation has been conducted. The obtained data at one point, i.e. 4 components, are used for the comparison between the computed and observed results. Complete agreement between the computed and observed results was obtained. Thus, the adaptability of the reduced Kalman filter finite element method has been shown. The present method is practically useful because the computational time can be drastically reduced.

2 THE KALMAN FILTER

Natural phenomena can generally be expressed by the following equation

\[ x_{k+1} = f_k(x_k) + G_kw_k \]  \hspace{1cm} (1)

where \( x_k \) denotes state variable at time \( t_k \), \( f_k(x_k) \) means transition function which is usually nonlinear function of \( x_k \), \( G_k \) is driving matrix, and \( w_k \) is system noise which may be included at the occasion of system modeling.
The observation $y_k$ can be obtained at some limited observation points and can be described as follows.

$$y_k = H_k x_k + v_k$$  \hspace{1cm} (2)$$

where $H_k$ means observation correspondence matrix which shows the relation between the observation and state variable and $v_k$ is the observation noise. Both $w_k$ and $v_k$ are assumed as:

$$w_k \sim N(0, Q)$$  \hspace{1cm} (3)$$

$$v_k \sim N(0, R)$$  \hspace{1cm} (4)$$

where $N(a,A)$ represents the white noise with the normal distribution of mean $a$ and covariance $A$. It is also assumed that

$$E\{w_k, v_j\} = 0$$  \hspace{1cm} (5)$$

where $E\{ \}$ is an expectation operator. Observation data are obtained up to time $t_k$ as:

$$Y_k = [y_1, y_2, \ldots, y_k].$$  \hspace{1cm} (6)$$

Taylor series expansions of $f_k(x)$ leads the following equation:

$$f_k(x_k) \simeq f_k(x_{k-1}) + D_k(x_k - x_{k-1}) + \cdots$$  \hspace{1cm} (7)$$

where

$$D_k = \left( \frac{\partial f_k}{\partial x_k} \right)$$  \hspace{1cm} (8)$$

Ignoring higher order terms and rearranging, it is obtained that

$$x_{k+1} = D_k x_k + B_k + G_k w_k$$  \hspace{1cm} (9)$$

where

$$B_k = f_k(x_{k-1}) - D_k x_{k-1}. \hspace{1cm} (10)$$

Normally eq.(9) is the linearized equation. However, using the iterations at time $t_k$, the nonlinear computation can be performed. The optimal estimate is an expectation of $x_k$, giving the observation data $Y_k$, which can be expressed as:

$$\hat{x}_{k/k} = E\{x_k \mid Y_k\}. \hspace{1cm} (11)$$

The initial condition is given

$$\hat{x}_0 = \hat{v}_0$$  \hspace{1cm} (12)$$

where $\hat{v}_0$ is a specified value at the initial time $t_0$. The state estimator $\hat{x}_{k/k}$ using the new measurement $y_k$ is of the form:

$$\hat{x}_{k/k} = \hat{x}_{k/k-1} + K_k [y_k - H_k \hat{x}_{k/k-1}]$$  \hspace{1cm} (13)$$

where $K_k$ is referred to as the Kalman gain, which will be given later in eq.(21), and $\hat{x}_{k/k-1}$ is a priori estimate,

$$\hat{x}_{k/k-1} = E\{x_k \mid Y_{k-1}\}. \hspace{1cm} (14)$$
The covariance $P_k$ is defined and derived as:

$$P_k = E\{ (x_k - \hat{x}_{k/k})(x_k - \hat{x}_{k/k})^T \}$$
$$= (I - K_k H_k) \Gamma_k$$

(15)

where covariance $\Gamma_k$ is defined as:

$$\Gamma_k = E\{ (x_k - \hat{x}_{k/k-1})(x_k - \hat{x}_{k/k-1})^T \}$$

(16)

and initial condition is

$$\Gamma_0 = \hat{\Gamma}_0$$

(17)

where $\hat{\Gamma}_0$ is a specified value at the initial time $t_0$. One step ahead prediction of the state variable is obtained as:

$$\hat{x}_{k+1/k} = D_k \hat{x}_{k/k} + \hat{B}_k$$

(18)

$$\hat{B}_k = f_k(x_{k/k}) - D_k \hat{x}_{k/k}$$

(19)

then, it is found that

$$P_{k+1} = D_k \Gamma_k D_k^T + Q.$$ 

(20)

The Kalman gain can be determined by

$$K_k = \Gamma_k H_k^T (R + H_k \Gamma_k H_k^T)^{-1}$$

(21)

and the covariance $\Gamma_k$ can be derived as follows:

$$\Gamma_{k+1} = D_k P_k D_k^T + G_k Q_k G_k^T.$$ 

(22)

3 REDUCED KALMAN FILTER FINITE ELEMENT METHOD

For the computation of the conventional Kalman filter finite element method, a large amount of computational load is needed. To reduce the load, the computational domain is divided into two parts, one is referred to as the main domain and the other is as the subsidiary domain. The main domain consists of observation points, estimation points and their related points. At the observation points, the state variables, such as velocity, water elevation, contaminant concentration are measured using physical instruments. For the estimation points, arbitrary points can be used at which the estimation of state variables can be obtained. If all nodal points in the whole computational domain are taken as those in the main domain, the estimation at the whole domain can be computed. In this research, a state variable at a nodal point is assumed to be non correlated with those at nodal points located far from itself. For the state variables in the normal water area, this assumption is valid as is previously discussed by authors group (Suga · Kawahara (2003), Yonekawa · Kawahara (2003)). The maximum length between the considering point and the correlated points is denoted by $dm$. The points inside the circle of radius $dm$ are referred to as the related points. The main domain includes the nodal points, at which the Kalman filter is applied. Contrary to this, the subsidiary domain is to introduce to carry out the finite element computation. At nodal variables in the subsidiary domain, the Kalman filter is not applied. The
spatial distribution of the covariance $S$ at the point with distance $r_i$ is assumed to be expressed by the quartic spline function as

$$S = \bar{S} \{1.0 - 6.0 \left( \frac{r_i}{d_m} \right)^2 + 8.0 \left( \frac{r_i}{d_m} \right)^3 - 3.0 \left( \frac{r_i}{d_m} \right)^4 \}$$

(23)

where $S$ stands for $Q$ and $R$, and $\bar{S}$ is the intensity of the covariance. The observation data obtained by the normal measurements always show the spatial distribution shown in eq.(23). This fact is already discussed and verified by Yonekawa · Kawahara (2003). The schematic view of the distribution of the covariance is shown in Figure 1. The subsidiary domain includes all nodal points other than those in the main domain.

Let $x_k$ be the state variable at nodal points in the main domain and $z_k$ be in the subsidiary domain, the transition matrix can be divided into four parts and the system equation eq.(9) can be described as follows

$$\begin{align*}
\begin{bmatrix}
    x_{k+1} \\
    z_{k+1}
\end{bmatrix} =
\begin{bmatrix}
    F_k & L_k \\
    M_k & N_k
\end{bmatrix}
\begin{bmatrix}
    x_k \\
    z_k
\end{bmatrix} +
\begin{bmatrix}
    U_k \\
    V_k
\end{bmatrix} +
\begin{bmatrix}
    G w_k \\
    0
\end{bmatrix}.
\end{align*}$$

(24)

Namely, the stochastic process can be applied to the state variable $x_k$ in the main domain only. The state variable $z_k$ in the subsidiary domain is used for auxiliary computation by the finite element method. Thus, one step ahead prediction of the state variable in the main domain can be derived as

$$\hat{x}_{k+1} = F_k \hat{x}_k + C_k$$

(25)
where the estimator in the main domain is denoted by \( \hat{x}_{k+1} \), \( F_k \) is the transition matrix expressed in eq.(24) and \( C_k \) is the known term and expressed as follows

\[
C_k = L_k(M_{k-1}x_{k-1} + N_{k-1}z_{k-1} + V_{k-1}) + U_k.
\]

(26)

The covariances \( P_{k+1} \) and \( \Gamma_{k+1} \) can be computed by the same equations as in eqs. (20) and (22), regarding \( \hat{x}_k \) as the optimal estimation in the main domain.

4 FINITE ELEMENT METHOD

4.1 State Equation

In this section, the indecial notation and summation convention with repeated indices are used. As the state equation, the non-linear shallow water equation and the advection diffusion equation are utilized. The coordinate system is shown in Figure 2.

4.1.1 Non-Linear Shallow Water Equation

The momentum and continuity equations in the non-linear shallow water equation are expressed as:

\[
\begin{align*}
\dot{u}_i + u_j u_{i,j} + g(\eta + H + h)_{,i} - \nu(u_{i,j} + u_{j,i})_{,j} &= 0 \\
\dot{\eta} + ((\eta + H)u_i)_{,i} &= 0
\end{align*}
\]

(27)

(28)

where \( u_i \) is water velocity in the eastward \( (x) \) and northward \( (y) \) directions, \( \eta \) is water elevation, \( g \) is the gravitational acceleration, \( \nu \) is kinematic eddy viscosity, \( h \) is bed elevation and \( H \) is water depth.

On the boundary \( \Gamma \), boundary conditions are given as follows:

\[
\begin{align*}
u_i &= \hat{\nu}_i \quad \text{on} \quad \Gamma_u, \\
u_n &= u_i n_i = \hat{u}_n \quad \text{on} \quad \Gamma_n, \\
\eta &= \hat{\eta} \quad \text{on} \quad \Gamma_\eta
\end{align*}
\]

(29)

(30)

(31)

where \( \hat{\nu}_i \) and \( \hat{\eta} \) mean the specified value on the boundary and \( n_i \) denotes unit outward normal to the boundary, \( \hat{u}_n \) is the specified velocity normal to the boundary.

Initial conditions are given as follows

\[
\begin{align*}
u_i &= \hat{\nu}_i^0 \quad \text{on} \quad t = 0, \\
\eta &= \hat{\eta}^0 \quad \text{on} \quad t = 0
\end{align*}
\]

(32)

(33)

Fig. 2: coordinate system
4.1.2 Advection Diffusion Equation

For contaminant concentration $c$, the following advection diffusion equation is utilized.

\[
\dot{c} + u_i c_i - \kappa c_{ii} = 0 \quad (34)
\]

where $\kappa$ is turbulent diffusion coefficient.

On the boundary $\Gamma$, boundary conditions are given as follows:

\[
c(t) = \hat{c}, \quad \text{on} \quad \Gamma_c, \quad (35)
\]

\[
b(t) = \kappa c_n n_i = \hat{b}, \quad \text{on} \quad \Gamma_n \quad (36)
\]

where $\hat{c}$ means the specified value on the boundary and $n_i$ denotes the unit outward normal to the boundary and $\hat{b}$ is the specified flux of concentration. Initial conditions are given as follows:

\[
c(t_0) = \hat{c}^0, \quad \text{on} \quad t = 0, \quad (37)
\]

4.2 Finite Element Equation

The discretization in space is carried out based on the finite element method. The velocity, water elevation and contaminant concentration are discretized on a triangular element using a linear interpolation function. For the weighting function, the same linear function is used. The explicit Euler method is applied to the discretization in time. The finite element equation denoting $u_{\alpha\beta}^{n+1}$, $\eta^{n+1}$, $c^{n+1}_{\alpha\beta}$ velocity, water elevation, contaminant concentration at nodal point at time $n$, respectively, can be derived as follows

\[
\tilde{M}_{\alpha\beta\gamma}^{n+1} u_{\beta\gamma}^{n+1} = \tilde{M}_{\alpha\beta\gamma} u_{\beta\gamma}^{n} - \Delta t (K_{\alpha\beta\gamma}^{n} u_{\beta\gamma}^{n} + D_{\alpha\beta\gamma}^{n} + H_{\alpha\beta\gamma}^{n} + \hat{\Omega}_{\alpha\beta}^{n}) \quad (38)
\]

\[
\tilde{M}_{\lambda\mu}^{n+1} \eta_{\mu}^{n+1} = \tilde{M}_{\lambda\mu} \eta_{\mu}^{n} - \Delta t (A_{\lambda\mu\beta}^{n} u_{\beta}^{n} + B_{\lambda\mu\beta}^{n} + \tilde{\Psi}_{\lambda\mu}^{n}) \quad (39)
\]

\[
\tilde{M}_{\alpha\beta\gamma}^{n+1} c^{n+1}_{\beta\gamma} = \tilde{M}_{\alpha\beta\gamma} c^{n}_{\beta\gamma} - \Delta t (K_{\alpha\beta\gamma}^{n} c^{n}_{\beta\gamma} + D_{\alpha\beta\gamma}^{n} + \hat{\Psi}_{\alpha\beta}^{n}) \quad (40)
\]

where $\tilde{M}_{\alpha\beta\gamma}$ is the lumped coefficient and $\tilde{M}_{\alpha\beta\gamma}$ is

\[
\hat{M}_{\alpha\beta\gamma} = e \tilde{M}_{\alpha\beta\gamma} + (1 - e) M_{\alpha\beta\gamma} \quad (41)
\]

in which $e$ is the lumping parameter adjusting the stability of computation. The coefficients in eqs.(38) ~ (41) can be written as follows:

\[
M_{\alpha\beta\gamma} = \int_V (\Phi_{\alpha} \Phi_{\beta}) \delta_{ij} dV \quad K_{\alpha\beta\gamma} = \int_V (\Phi_{\alpha} \Phi_{\beta} \Phi_{\gamma,j}) dV \quad H_{\alpha\beta\gamma} = g \int_V (\Phi_{\alpha,i} \Phi_{\lambda}) dV
\]

\[
D_{\alpha\beta\gamma} = \nu \int_V (\Phi_{\alpha,j} \Phi_{\beta,i}) dV + \nu \int_V (\Phi_{\alpha,k} \Phi_{\beta,k}) \delta_{ij} dV \quad \hat{\Omega}_{\alpha\beta} = g H_{\alpha\beta\gamma} (H_\mu + h_\mu)
\]

\[
M_{\lambda\mu} = \int_V (\Phi_{\lambda} \Phi_{\mu}) dV \quad A_{\lambda\mu\beta} = \int_V (\Phi_{\lambda} \Phi_{\mu} \Phi_{\beta,j}) dV + \int_V (\Phi_{\lambda} \Phi_{\beta} \Phi_{\mu,j}) dV \quad B_{\mu\beta} = A_{\lambda\mu\beta} (H_\mu + h_\mu) \quad D_{\alpha\beta} = \kappa \int_V (\Phi_{\alpha,k} \Phi_{\beta,k}) dV
\]

where $H_\mu$ and $h_\mu$ are the water depth and bed elevation at nodal point, respectively.
4.3 Algorithm

The transition matrix in eq.(24) can be derived from eqs. (38)-(41) as follows

\[
\begin{pmatrix}
u \\
v \\
\eta \\
c
\end{pmatrix}^{k+1} = [D_k]
\begin{pmatrix}
u \\
v \\
\eta \\
c
\end{pmatrix}^{k} + \{A_k\} + \{M_k\}
\]

(42)

where \([D_k]\) is referred to as the transition matrix and \(\{A_k\}\) is the known term. Dividing the matrix \([D_k]\) into four parts as shown in eq.(24), matrix \([F_k]\) can be obtained.

The algorithm of the reduced Kalman filter finite element method is written as follows.

1. \(\Gamma_0, \{\hat{x}_0\} : \text{given}\)
2. \(K_k = [\Gamma_k][H_k]^T ([R] + [H_k][\Gamma_k][H_k]^T)^{-1}\)
3. \(P_k = ([I] - [K_k][H_k])[\Gamma_k]\)
4. \(\Gamma_{k+1} = [F_k][P_k][F_k]^T + [Q][G_k][Q]^T\)
5. \(\{\hat{x}_{k/k-1}\} = [F_k]\{\hat{x}_{k-1/k-1}\} + \{M_k\}\)
6. \(\{M_k\} = \{A_{k-1}\} + \{C_k\}\)
7. \(\{C_k\} = L_k(M_{k-1}\hat{x}_{k-1} + N_{k-1}z_{k-1} + V_{k-1}) + U_k\)
8. \(\{\hat{x}_{k/k}\} = \{\hat{x}_{k/k-1}\} + [K_k]\{y_k\} - [H_k]\{\hat{x}_{k/k-1}\}\)

where \(\hat{x}_{k/k}\) is an estimation of water velocity, water elevation and contaminant concentration in the main domain at time \(k\) and \(z_k\) is variables in the corresponding subsidiary domain, which can be obtained by the second half of eq.(24), respectively.

5 Application to the Teganuma River

The Teganuma river is located in Chiba prefecture in Japan, of which photograph is shown in Figure 3. The total length of the computed area is 4km and the maximum depth is 2.5m. The observed data, velocity, water elevation and contaminant concentration, in the river is obtained on August 16th in 2003 conducted by the Teganuma Down Stream River Office, the Ministry of Land, Infrastructure and Transport, the Government of Japan. The water depth is shown in Figure 4. The four observation points are illustrated in Figure 5.
Fig. 3: Teganuma river

Fig. 4: Water Depth
There are four observed points at the area to be analyzed in the Teganuma river. The main domain used in the computation is shown in Figure 5. The black dot • means the observed points and locations are in the upper figure. The white dot ○ denotes the related points. Altogether 28 points are used as the main domain.

5.1 Observed Data

The eastward and northward coordinates are denoted by $x$ and $y$. The observed velocities in the $x$ and $y$ directions, water elevations, and the observed DO concentration which is the index of contaminant at observation point No.1 are plotted in Figures 6-9. The maximum velocity is 0.025m/s in the $x$ direction and 0.02m/s in the $y$ direction. The maximum change of water elevation is 0.1m. The maximum concentration is 6.7 ppm.
5.2 Finite Element Computation

The finite element mesh is represented in Figure 10. The total number of nodes and elements are 1316 and 2378, respectively. Specifying velocity, water elevation and contaminant concentration at the observation points Nos.1, 2 and 4, the steady state computation is carried out to obtain the initial condition. The velocity distribution at the initial state is expressed in Figure 11. The contaminant concentration distribution at the initial state is expressed in Figure 12. For the observation, four components of velocity, water elevation and contaminant concentration at three points are used. For the estimation computation, two components of velocity, water elevation and contaminant concentration are used. Time increment $\Delta t = 1.0$ and lumping parameter $e = 0.9$ are employed kinematic eddy viscosity $\nu = 0.0025$ and turbulent diffusion coefficient is $\kappa = 1.5$. Quartic spline functions with the intensities of $\bar{R} = 0.001$ and $\bar{Q} = 0.001$ are used for observation and system error covariances $R$ and $Q$. Starting $\Gamma_0$ equals 0.01 $E$ where $E$ is unit matrix, the computation based on the main domain is carried out.

![Fig. 10: Finite Element Mesh](image)

![Fig. 11: Initial velocity used for the computation](image)
Fig. 12: Initial contaminant concentration used for the computation

5.3 Computational Results

Specifying velocity, water elevation and contaminant concentration at the observation points Nos.1, 2 and 4, the steady state computation is carried out to obtain the initial condition. The main domain consists of the nodes inside the circles of which centers are observation and estimation points as shown in Figure 13. For the observation, two components of velocity, water elevation and contaminant concentration at Nos.1, 2 and 4 are used. The computed velocity, water elevation and contaminant concentration at No.3 are compared and represented in Figures 14 ∼ 17. Red and blue lines show the estimated and observed results, respectively. The average discrepancy of velocity is within $1 \sigma$ in the $x$ direction in Figure 14, where $\sigma$ is the corresponding square root of the diagonal term of $P_k$, and $1 \sigma$ in the $y$ direction in Figure 15. The average discrepancy of water elevation is within $1 \sigma$ in Figure 16. The average discrepancy of contaminant concentration is within $2 \sigma$ in Figure 17. Both observed and estimated results are quite well in agreement. Velocity and contaminant concentrations at time 7 hours passed from the initial time are shown in Figures 18 and 19.

Fig. 13: Observation points Nos.1,2 and 4 marked by ○ and estimation point at No.3 marked by ●
Fig. 14: Comparison of X-velocity at No.3 between observation and estimation

Fig. 15: Comparison of Y-velocity at No.3 between observation and estimation
Fig. 16: Comparison of Water Elevation at No.3 between observation and estimation

Fig. 17: Comparison of Dissolved Oxygen at No.3 between observation and estimation
Fig. 18: Velocity distribution at time 7 hours passed

Fig. 19: Initial contaminant concentration at time 7 hours passed

6 CONCLUSION

The new approach to the Kalman filter finite element method is presented in this paper. The computational capacity and calculation time can be reduced by the present method, assuming that the state variables in the subsidiary domain are not correlated with those in the main domain. The Kalman filter is applied only to the state variables in the main domain. Computed velocity, water elevation and contaminant concentration obtained in the Teganuma river are in close agreement with the observed results. The distribution of the observation points should be wide spreaded all over the domain for computation. The present Kalman filter finite element method is shown to be a useful tool for the estimation of velocity, water elevation and contaminant concentration.
References


