An Identification Method of Geological Boundaries Using First Order Adjoint Equation

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Abstract

In this paper, a technique for identifying boundaries of the ground strata in front of a tunnel excavation site using the first order adjoint method based on the optimal control theory is proposed. At tunnel excavations, it is important to presume the ground situation ahead of the cutting face beforehand. Excavating into weak strata or fault fracture zones may cause extension of the construction work and human suffering. A theory for determining positions of the geological boundaries in a numerical manner is investigated, employing excavating blasts and its vibration waves as the observation references. According to the optimal control theory, the performance function described by the square sum of the residuals between computed and observed velocities is minimized. The elastic analysis governed by the Navier equation is carried out, assuming the ground as an elastic body with linear viscous damping. To identify the geological boundaries, the gradient of the performance function with respect to the coordinates of boundaries can be calculated using the adjoint equation. The weighed gradient method is effectively applied to the minimization algorithm. To solve the governing and adjoint equations, the Galerkin finite element method and the average acceleration method are employed for the spatial and temporal discretizations, respectively. Based on the method presented in this paper, the boundary layers located between two strata of different elastic moduli can be identified. For the numerical studies, the Kasakura tunnel excavation site is employed.

Key words: Parameter identification, Finite element method, Average acceleration method, First order adjoint equation method, Weighted gradient method, Geological boundaries, Navier equation, Optimal control theory.

1 Introduction

It is highly important to understand in advance behavior and characteristics of the ground in the civil engineering works such as tunnels, traffic roads, dams, etc. To know what kind of behavior the externally forced ground shows is directly related to safety managements, cost reduction measures and environmental assessments, etc., in the constructions. Until now, the property investigations of the ground such as geological reconnaissance of a drilling survey, geophysical exploration, paling of investigation and rock test have been used as the generalized method in designing the civil engineering works. However, the conventional investigative approaches require a lot of time and cost for investigation, and sometimes it leads that the construction must be prolonged for the investigation. Thus, a number of improvements should be developed for these reasons. Consequently, the forecast technique with a numerical simulation, by which the constructions can be carried out more safer and less expensive, is considerably developed by the progress of a computer and the related analytical technology recently.
At the tunnel constructions, the problem that excavating into weak strata or fault fracture zones without careful preparation sometimes may cause prolongation of the construction works and human damages. Thus, in this research, an identification technique is presented to examine in a numerical manner the ground boundary of two geological strata. On the assumption that a tunnel is excavated, the positions of the ground boundary are identified using ground vibration caused by blasts with dynamite and the first order adjoint equation method. It is possible that useful information can be obtained beforehand for going into the weak ground strata and fault fracture zones in the excavating works by identifying the geological boundary layers. If not only data on elastic coefficients but also boundary positions in the geological strata can be explored, safer and less expensive excavation works are possible. Moreover, this technique can be performed without stopping the excavation works at the cutting face consuming less expensive search costs.

Identification of the position of the geological boundaries considered becomes possible by applying the present technique. The results of this research leads to a plenty of contribution to the engineering works because the position of the geological boundary layers can be determined by this technique. A number of methods for parameter identification have been presented in the fields of meteorology e.g. [2] and hydraulic mechanics e.g. [6] or [12]. Methods for identification of parameters such as seepage and temperature of the ground have been discussed by Asai and Kawahara [8], Kojima et al. [4], Kawahara et al. [5]. To estimate the geological structure in the ground, an identification method of rock parameters have been presented by Chaparro et al. [1], Swoboda et al. [3], Koizumi and Kawahara [7], Ohkami and Swoboda [9], Huang and Liu [11], Xiang et al. [13]. However, it seems that few attempt in the technique for particularly identifying the geological boundaries exist though a number of approach of inverse analyses have been proposed. This research becomes one of the contributions to the technique for identifying the geological boundary layers.

The present method is applied to the rectangular computational model for verification. After the verification, the excavation site at Kasakura tunnel is identified. In this site, the geological boundaries has been determined by using the observed velocities measured on the top of the mountain based on vibrations caused by the actual blast.

Figure 1: Sectional view of tunnel excavation

2 Basic Equation

In this paper, the indecial notation and summation convention are used to describe equations. The geological boundaries are identified by the following theories. The natural ground is composed of some strata as shown in Figure 1.

The Navier equation is applied to a dynamic analysis of the ground as a governing equation. In this research, the ground is assumed as a linear elastic body. The governing equation can be expressed as follows;

\[ D_{ijkl} u_{k,lj} + \rho b_i - \rho \ddot{u}_i = 0, \]

(1)

where \( u_i \), \( b_i \) and \( \rho \) denote displacement, body force and density of the ground, respectively. Here, over dot denotes time differentiation. \( D_{ijkl} \) is called elastic coefficient matrix, and defined by the following expressions;
\[ D_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}), \]  

(2)

where \( \delta_{ij} \) is the Kronecker delta, \( \lambda \) and \( \mu \) are the Lame's constants and can be written as follows;

\[ \lambda = \frac{\nu E}{(1 - 2\nu)(1 + \nu)}, \]  

(3)

\[ \mu = \frac{E}{2(1 + \nu)}, \]  

(4)

in which \( E \) and \( \nu \) are the elastic modulus and the Poisson ratio, respectively. Based on the elastic modulus and the Poisson ratio of each stratum, \( D_{ijkl} \) can be derived as follows;

\[ D_{ijkl} = \sum_{m=1}^{M} D^{(m)}_{ijkl}, \]  

(5)

where \( M \) is the maximum number of strata and \( D^{(m)}_{ijkl} \) is elastic coefficient matrix at stratum \( m \).

The basic equation is solved based on the boundary and the initial conditions. The boundaries \( \Gamma_1 \) and \( \Gamma_2 \) are known boundary of displacement \( u_i \) and surface force \( t_i \), respectively;

\[ u_i = \hat{u}_i \quad \text{on} \quad \Gamma_1, \]  

(6)

\[ t_i = D_{ijkl} u_k n_j = \hat{t}_i \quad \text{on} \quad \Gamma_2, \]  

(7)

where \( \hat{u}_i \) and \( \hat{t}_i \) mean the known displacement and surface force, \( n_j \) represents direction cosine of the unit normal diverted to the outside on the boundary, respectively. The total boundary \( \Gamma \) should satisfy the following condition;

\[ \Gamma_1 \cup \Gamma_2 = \Gamma, \]  

\[ \Gamma_1 \cap \Gamma_2 = \emptyset. \]

The initial conditions are given as follows;

\[ u_i = \hat{u}_i^0 \quad \text{at} \quad t = t_0, \]  

(8)

\[ \dot{u}_i = \dot{\hat{u}}_i^0 \quad \text{at} \quad t = t_0, \]  

(9)

where \( \hat{u}_i^0 \) and \( \dot{\hat{u}}_i^0 \) are the specified values at the initial stage.

3 Discretization Techniques

3.1 Finite Element Method

As for the spatial discretization, the finite element method is applied. A computational domain is divided into triangular elements, and approximated by the linear polynomial of coordinates \( x \) and \( y \). The finite element equation can be expressed as follows;

\[ M_{\alpha i\beta k} \ddot{u}_{\beta k} + K_{\alpha i\beta k} u_{\beta k} = \hat{\Gamma}_{\alpha i}, \]  

(10)

Considering the effect of damping, eq.(10) can be transformed into the following equation;

\[ M_{\alpha i\beta k} \ddot{u}_{\beta k} + C_{\alpha i\beta k} \dot{u}_{\beta k} + K_{\alpha i\beta k} u_{\beta k} = \hat{\Gamma}_{\alpha i}, \]  

(11)
where damping and the other coefficient matrices are written as follows;

\[
M_{\alpha i \beta k} = \int_V \rho \delta_{ik} N_\alpha N_\beta dV, \\
K_{\alpha i \beta k} = \int_V N_{\alpha,j} D_{ijkl} N_{\beta,l} dV, \\
C_{\alpha i \beta k} = \alpha_0 M_{\alpha i \beta k} + \alpha_1 K_{\alpha i \beta k}, \\
\hat{\Gamma}_{\alpha i} = \int_{\Gamma} \Gamma_2 N_\alpha \hat{t}_i d\Gamma + \int_V \rho N_\alpha \hat{b}_i dV,
\]

where \( M_{\alpha i \beta k} \), \( K_{\alpha i \beta k} \), and \( \hat{\Gamma}_{\alpha i} \) are mass, stiffness, and load matrices, respectively, in which \( N_\alpha \) is called as the shape function, which expresses the approximate geometry of displacement distribution. For the damping matrix \( C_{\alpha i \beta k} \), Rayleigh damping defined by the sum of the damping proportional to the mass and proportional to the stiffness is introduced. Two coefficients \( \alpha_0 \) and \( \alpha_1 \) in eq.(14) are parameters obtained by the characteristic frequency of elastic body and the damping constant.

### 3.2 Average Acceleration Method

As for the temporal discretization, the average acceleration method is applied to the finite element equation. The average acceleration method is a numerical technique to solve the second order differential equation. It is assumed that the acceleration at \( t^{(n)} \leq t \leq t^{(n+1)} \) is equal to the mean value of the acceleration of \( t^{(n)} \) and \( t^{(n+1)} \), and constant. In the average acceleration method, the displacement and the velocity at \( (n+1) \) time are assumed as follows;

\[
u^{(n+1)}_{\beta k} = \nu^{(n)}_{\beta k} + \Delta t \dot{\nu}^{(n)}_{\beta k} + \frac{\Delta t^2}{4} (\ddot{\nu}^{(n+1)}_{\beta k} + \ddot{\nu}^{(n)}_{\beta k}), \\
\dot{\nu}^{(n+1)}_{\beta k} = \dot{\nu}^{(n)}_{\beta k} + \frac{\Delta t}{2} (\ddot{\nu}^{(n+1)}_{\beta k} + \ddot{\nu}^{(n)}_{\beta k}),
\]

where \( \Delta t \) is time increment, \( \nu^{(n+1)}_{\beta k} \) and \( \dot{\nu}^{(n+1)}_{\beta k} \) denote the displacement and the velocity at \( (n+1) \) time, respectively. These equations are identical to the Newmark \( \beta \) method with \( \gamma = 1/2 \) and \( \beta = 1/4 \). The finite element equation at time \( (n+1) \) is expressed as follows denoting quantities in the present time as \( (n) \);

\[
M_{\alpha i \beta k} \ddot{\nu}^{(n+1)}_{\beta k} + C_{\alpha i \beta k} \dot{\nu}^{(n+1)}_{\beta k} + K_{\alpha i \beta k} \nu^{(n+1)}_{\beta k} = \hat{\Gamma}_{\alpha i},
\]

Substituting eqs.(16) and (17) into eq.(18), we derive the following equation;

\[
D_{\alpha i \beta k} \ddot{\nu}^{(n+1)}_{\beta k} = \hat{\Gamma}_{\alpha i} - E_{\alpha i \beta k} \dot{\nu}^{(n)}_{\beta k} - F_{\alpha i \beta k} \nu^{(n)}_{\beta k} - K_{\alpha i \beta k} \nu^{(n)}_{\beta k},
\]

where each matrices are written as follows;

\[
D_{\alpha i \beta k} = M_{\alpha i \beta k} + \frac{\Delta t}{2} C_{\alpha i \beta k} + \frac{\Delta t^2}{4} K_{\alpha i \beta k}, \\
E_{\alpha i \beta k} = \frac{\Delta t}{2} C_{\alpha i \beta k} + \frac{\Delta t^2}{4} K_{\alpha i \beta k}, \\
F_{\alpha i \beta k} = C_{\alpha i \beta k} + \Delta t K_{\alpha i \beta k},
\]

Calculating acceleration \( \ddot{\nu}^{(n+1)}_{\beta k} \) using eq.(19) and substituting these into eqs.(16) and (17), displacement \( \nu^{(n+1)}_{\beta k} \) and velocity \( \dot{\nu}^{(n+1)}_{\beta k} \) can be obtained.
4 Performance Function

The purpose of this paper is to identify the geological boundaries located between various strata of different elastic moduli. To solve this type of inverse problem, the performance function should be introduced, which consists of the square sum of discrepancies between the computed and observed velocities. For the observed velocities, the measured values at the observation points are used. The parameter identification problem is performed by finding appropriate boundary coordinates of geological stratum so as to minimize the following performance function;

\[
J = \frac{1}{2} \int_{t_0}^{t_f} (\dot{u}_{\alpha i} - \dot{u}_{\alpha i}^*) W_{\alpha i\beta k}(\ddot{u}_{\beta k} - \ddot{u}_{\beta k}^*) dt, \tag{23}
\]

where \(\dot{u}_{\alpha i}\) and \(\dot{u}_{\alpha i}^*\) are the computed and the observed velocities at observed point \(\alpha\) in the \(i\)-direction, \(W_{\alpha i\beta k}\) is the diagonal matrix adjusting dimensions of the observed data, \(t_0\) and \(t_f\) are the initial and the final times, respectively.

5 First Order Adjoint Equation

The extended performance function is introduced, which can be expressed as follows;

\[
J^* = \frac{1}{2} \int_{t_0}^{t_f} (\dot{u}_{\alpha i} - \dot{u}_{\alpha i}^*) W_{\alpha i\beta k}(\ddot{u}_{\beta k} - \ddot{u}_{\beta k}^*) dt + \int_{t_0}^{t_f} \lambda_{\alpha i}(\dddot{u}_{\alpha i} - M_{\alpha i\beta k}\ddot{u}_{\beta k} - C_{\alpha i\beta k}\dot{u}_{\beta k} - K_{\alpha i\beta k}u_{\beta k}) dt, \tag{24}
\]

where \(\lambda_{\alpha i}\) is the Lagrange multipliers, which provide a strategy for the mathematical optimization with constraints to that without constraints. In the inverse analysis, it is necessary to calculate the gradient of the performance function to update an unknown variables. Using the gradient, the objective coordinate values of the boundaries are updated by the iterative calculation. The gradient of the performance function, the adjoint equation, and the terminal conditions can be obtained by taking the first variation of eq.(24) as follows;

\[
\delta J^* = \int_{t_0}^{t_f} (\dot{u}_{\alpha i} - \dot{u}_{\alpha i}^*) W_{\alpha i\beta k}\delta\ddot{u}_{\beta k} dt + \int_{t_0}^{t_f} \delta\dot{\lambda}_{\alpha i}\dddot{u}_{\alpha i} - M_{\alpha i\beta k}\ddot{u}_{\beta k} - C_{\alpha i\beta k}\dot{u}_{\beta k} - K_{\alpha i\beta k}u_{\beta k} dt - \int_{t_0}^{t_f} \lambda_{\alpha i}(M_{\alpha i\beta k}\delta\ddot{u}_{\beta k} + C_{\alpha i\beta k}\delta\dot{u}_{\beta k} + K_{\alpha i\beta k}\delta u_{\beta k}) dt + \int_{t_0}^{t_f} \lambda_{\alpha i}B_{\alpha i\beta k}^{(m)}\delta x_{\beta k} dt, \tag{25}
\]

where \(x_{\beta k}\) is the coordinates of positions of the boundary \(\Gamma^{(m)}\) and \(B_{\alpha i\beta k}^{(m)}\) is formulated as;

\[
B_{\alpha i\beta k}^{(m)} = \int_{\Gamma^{(m)}} (N_{\alpha}D_{ijkl}^{(m)}N_{\beta,l})n_j d\Gamma. \tag{26}
\]

Integrating by parts, first variation of the extended performance function is transformed into the following form;

\[
\delta J^* = (\ddot{u}_{\alpha i}(t_f) - \ddot{u}_{\alpha i}^*(t_f)) W_{\alpha i\beta k}\delta\ddot{u}_{\beta k}(t_f) - (\dot{u}_{\alpha i}(t_0) - \dot{u}_{\alpha i}^*(t_0)) W_{\alpha i\beta k}\delta\ddot{u}_{\beta k}(t_0) - \lambda_{\alpha i}(t_f)M_{\alpha i\beta k}\delta\ddot{u}_{\beta k}(t_f) + \lambda_{\alpha i}(t_0)M_{\alpha i\beta k}\delta\ddot{u}_{\beta k}(t_0) + \dot{\lambda}_{\alpha i}(t_f)M_{\alpha i\beta k}\delta\dot{u}_{\beta k}(t_f) - \dot{\lambda}_{\alpha i}(t_0)M_{\alpha i\beta k}\delta u_{\beta k}(t_0) - \lambda_{\alpha i}(t_f)C_{\alpha i\beta k}\delta u_{\beta k}(t_f) + \lambda_{\alpha i}(t_0)C_{\alpha i\beta k}\delta u_{\beta k}(t_0), \tag{26}
\]
The stationary condition can be expressed as:

\[ \delta J^* = 0 \]  \hspace{1cm} (28)

Considering each term of eq.(27), the first order adjoint equation and the terminal conditions of the adjoint variables can be derived. The adjoint equation is expressed as follows;

\[ -\ddot{\lambda}_{ai}M_{aij} - \dot{\lambda}_{ai}C_{aij} - \lambda_{ai}K_{aij} - (\ddot{u}_{ai} - \ddot{u}_{ai}^*)W_{aij} = 0, \]  \hspace{1cm} (29)

and the terminal conditions for the adjoint equation is obtained as;

\[ \lambda_{ai}(t_f) = 0, \]  \hspace{1cm} (30)

\[ \dot{\lambda}_{ai}(t_f)M_{aij} + (\ddot{u}_{ai}(t_f) - \ddot{u}_{ai}^*(t_f))W_{aij} = 0, \]  \hspace{1cm} (31)

The terminal condition of the acceleration can be obtained by substituting eqs.(30) and (31) to eq.(29). The state equation (11) can be solved from initial time to final time. On the other hand, the terminal conditions are specified as eqs.(30) and (31), then, the adjoint equation can be integrated from final time to initial time.

6 Gradient with Respect to Boundary Position

The gradient of the extended performance function with respect to the position of geological boundary, \( \nabla (J^*)_{\beta k} \) can be derived in the following manner. Let \( \Gamma_t \) be the target boundary to be obtained, but, not known at the initial stage. Thus, assume the initial boundary, which is denoted by \( \Gamma_i \) as shown in Figure 2;

If the boundaries are given as in Figure 2, the gradient of position of the boundary \( \Gamma_i \) to move from the assumed position to the final position \( \Gamma_t \) can be defined. Considering the adjacent boundary between \( (m) \) and \( (m+1) \) strata, the gradient can be obtained on the boundary \( \Gamma^{(m)} \). If the initial boundary \( \Gamma_i^{(m)} \) is identical to the target boundary \( \Gamma_t^{(m)} \), then eq.(26) should be as;

\[ B_{aij}^{(m)} = 0, \]  \hspace{1cm} (32)
However, in general $\Gamma_i^{(m)}$ is not identical to $\Gamma_i^{(m)}$, therefore, $B^{(m)}_{\alpha\iota\beta k}$ is not equal to 0. In this case, $\delta J^*$ can be rewritten as follows:

$$
\delta J^* = \int_{t_0}^{t_f} \lambda_{\alpha i} B^{(m)}_{\alpha\iota\beta k} \delta x_{\beta k} dt,
$$

where $x_{\beta k}$ is the coordinates of the boundary $\Gamma_i^{(m)}$. Eq.(33) means:

$$
grad(J^*)_{\beta k} = \lambda^{(m)}_{\alpha i} B^{(m)}_{\alpha\iota\beta k},
$$

The gradient of the extended performance function $grad(J^*)_{\beta k}$ with respect to the position of the geological boundary coordinates $x_{\beta k}$ can be obtained as in eq.(34). Namely, the geological boundary is identified using the gradient expressed in eq.(34).

7 Weighted Gradient Method

The weighted gradient method is applied to the minimization technique in this research. The weighted gradient method has the comparatively simple algorithm, but the convergence of the method is stable. Denoting the coordinates of boundary $x_{\beta k}$ at $l$-iteration $x^{(l)}_{\beta j}$, the modified performance function can be expressed as follows:

$$
K^{(l)} = J^{* (l)} + \frac{1}{2} \int_{t_0}^{t_f} (x^{(l+1)}_{\alpha i} - x^{(l)}_{\alpha i}) W^{(l)}_{\alpha\iota\beta j} (x^{(l+1)}_{\beta j} - x^{(l)}_{\beta j}) dt,
$$

where $W^{(l)}_{\alpha\iota\beta j}$ is the adjusting weights to secure the stable computation. Taking the variation of eq.(35) with respect to $x^{(l+1)}_{\beta j}$, the optimal condition of the modified performance function can be expressed as follows;

$$
\delta K^{(l)} = 0.
$$

Considering the optimal condition (36), the following update formulation is derived;

$$
W^{(l)}_{\alpha\iota\beta j} x^{(l+1)}_{\beta j} = W^{(l)}_{\alpha\iota\beta j} x^{(l)}_{\beta j} - grad(J^{* (l)})_{\alpha i},
$$

The new positions of geological boundary $x^{(l+1)}_{\beta j}$ is obtained by using eq.(37).
8 Verification of Identification Method

By numerical study, the identification technique is verified by using the adjoint equation which is derived in the preceding sections. To verify availability of the method, which is an identification of the geological boundary, the simplified model is used. The computational model is simple and suitable for verification of the identification technique.

The finite element mesh is shown in Figure 3. The total number of nodes and elements are 861 and 1600, respectively. This mesh is used in all computations in the following cases 1 ∼ 3. The observed point is set to \((X, Y) = (1.0, 1.0)\). The momentary force is given at the force point on the right-hand boundary. In this paper, the actual geological boundary, that is the boundary of two strata with different elastic moduli, is identified assuming initial boundary, of which positions are different from the actual boundary. Boundary positions can be expressed by the coordinates, \(X\) and \(Y\). The actual boundary, which is the target boundary of the computation, is placed in advance. This is not known at the starting position. Thus, the initial boundary is assumed one. It is important what value is assumed at the initial boundary. To verify the general versatility, three cases which are different initial conditions are analyzed. The Poisson ratio, density of the ground and time increment are set to 0.3, \(2.3 \times 10^3[g/m^3]\) and \(1.0 \times 10^{-2}[sec]\), respectively. Two components of the ground velocity are computed at the observed point. Based on the two components of the computed velocity as the observation value, the identification of the geological boundary has been carried out. Elastic moduli used are \(E_1 = 1.0 \times 10^5[kN/m^2]\) in layer-1 and \(E_2 = 1.5 \times 10^5[kN/m^2]\) in layer-2, respectively. The \(u\) and \(v\) in Figure 4 mean displacements in the \(X\) direction and in the \(Y\) direction on the boundary, respectively.
8.1 Case 1: Vertical Boundary

In case 1, it is assumed that the geological boundary is located in a vertical direction on the ground. The initial and target geological boundaries are shown in Figures 5 and 6. \( X \) coordinate of the initial geological boundary is set at the position \( X = 1.5[m] \). \( X \) coordinate of the target geological boundary is set at the position \( X = 1.0[m] \).

![Figure 5: Initial assumed boundary](image1)

![Figure 6: Target boundary](image2)

Red points plotted in Figure 7 show the variation of the performance function. It is shown in Figure 7 that the performance function to be converged to 0. This result means that the calculated velocities are coincident with the observation velocities. Blue points represented in Figure 8 show that the movements of the geological boundaries from an initial to a target values. Figure 9 shows the identified boundary. The identified boundary is coincident with the target boundary.

![Figure 7: The variation of performance function](image3)

![Figure 8: The movement of boundary coordinate](image4)

![Figure 9: Identified boundary](image5)
8.2 Case 2: Inclined Boundary

In case 2, it is assumed that the geological boundary is located in an inclined direction on the ground. The initial and target geological boundaries are shown in Figures 10 and 11. The equation of the initial boundary is \( Y = X - 1[m] \). The equation of the target boundary is \( Y = X[m] \).

Red points plotted in Figure 12 show the variation of the performance function. It is shown that the performance function to be converged to 0. Blue points in Figure 13 show that the movements of the geological boundaries from an initial to a target values. Figure 14 shows the identified boundary. The identified boundary is coincident with the target boundary.
8.3 Case 3: Horizontal Boundary

In case 3, it is assumed that the geological boundary is located in a horizontal direction on the ground. The initial and target geological boundaries are shown in Figures 15 and 16. Y coordinate of the initial geological boundary is set at the position \( Y = 0.1[m] \). Y coordinate of the target geological boundary is set at the position \( Y = 0.5[m] \).

Red points in Figure 17 show the variation of the performance function. It is shown that the performance function to be converged to 0. Blue points represented in Figure 18 show that the movements of the geological boundaries from an initial to a target values. Figure 19 shows the identified boundary. The identified boundary is coincident with the target boundary.
9 Application to Kasakura Tunnel

This technique is applied to the Kasakura tunnel excavation site. The position of geological boundary is identified by using elastic waves caused by blasts at the top of the mountain.

9.1 Kasakura Tunnel

The Kasakura tunnel construction site is located in Fukui prefecture in Japan. The length of the tunnel is $425\text{m}$ and the excavated length at the time that the observation was taken place is about $305\text{m}$. The altitude of this mountain is about $342\text{m}$. Figures 20 and 21 show the planning map and vertical section, respectively. The computational area is shown in Figure 22. The length of computational area is $160\text{m}$. There are three geological strata in this area. These are called as stratum 1, 2 and 3.

![Planning map of Kasakura tunnel](image1)

Figure 20: Planning map of Kasakura tunnel

![Vertical section](image2)

Figure 21: Vertical section
9.2 Identification of Geological Boundary

The terrain data for generating the finite element mesh and rock properties are quoted by a geological investigative report of Kasakura tunnel. The finite element mesh is shown in Figure 23. The total number of nodes and elements are 7369 and 14457, respectively. The observed points are set to \((X_1, Y_1) = (43.2, 169.1)\) and \((X_2, Y_2) = (102.6, 149.6)\). The Borehole pressure which was introduced by U. S. Army Corps of Engineers [10] is given from the left-hand side at the tunnel face. The Poisson ratio and time increment are set to 0.32 and \(1.0 \times 10^{-3} [\text{sec}]\). Elastic moduli and densities of three strata are listed in Table 1. In this case, the geological boundary between strata 2 and 3 is determined, observing the elastic waves at the top and side place of the mountain. Figure 24 shows the computational conditions. The following Figures 25 ~ 28 are velocities measured at observation points. One is located small hill, which is named point No.1, and the other is at the middle place of the hill, which is point No.2. At the observation points, 30[cm] \times 30[cm] \times 5[cm] mortar bases are made, after 50[cm] depth of the surface soil is removed. The axis of coordinate for \(X\) is defined as not the north of the azimuth direction but the direction of movement in the tunnel excavation, and \(Y\) is vertical direction. Thus, the velocities in the \(X\) direction and in the \(Y\) direction were measured. These measure values are employed as the observation data for the computation.
Figure 25: Observed velocity in the X direction at point No.1

Figure 26: Observed velocity in the Y direction at point No.1

Figure 27: Observed velocity in the X direction at point No.2

Figure 28: Observed velocity in the Y direction at point No.2

The initial assumed geological boundary is shown in Figures 29. Starting from the geological boundary in Figure 29, and after 37 iterations, the identified boundary is obtained and shown in Figure 30. The identified boundary is almost coincident with the ones obtained by the pre-construction drilling tests as shown Figure 22.

Red points plotted in Figure 31 show the variation of the performance function. It is shown in Figure 31 that the performance function is converged to steady value. Blue points represented in Figure 32 show the movements of the geological boundaries from an initial to a final values. The movement of boundary coordinate at the top of the mountain is plotted in Figure 32. The computed boundary is coincident with the Investigated boundary.
The computed velocity at observation points No.1 and No.2 are compared with the observation velocity and shown in Figures 33 ~ 36. The observed and computed velocities are almost agree in any cases.

Figure 31: The variation of performance function
Figure 32: The movement of boundary coordinate

Figure 33: Comparison of velocity in the X direction at point No.1
Figure 34: Comparison of velocity in the Y direction at point No.1

Figure 35: Comparison of velocity in the X direction at point No.2
Figure 36: Comparison of velocity in the Y direction at point No.2
10 Conclusion

In this paper, an identification technique to determine the position of geological boundaries has been presented. The first order adjoint equation method of the optimal control theory was usefully used to identify the geological boundaries. Minimizing the performance function, which is the square sum of the discrepancies between the computed and observed velocities, the geological boundaries were identified. The weighted gradient method which uses the gradient derived by the adjoint equation method was applied as a minimization technique. Effectiveness of the technique has been shown using the verification models. The availability of the present method is confirmed by using the blasting velocity obtained the Kasakura tunnel. Safety and less expensive excavating works became possible by the establishment of the technique for the identification of positions of geological boundaries. As future research, this technique will be improved to do more accurate analysis. For example, the update technique of the finite element mesh and deriving theory of the gradient will be discussed. Then, it is necessary to apply the identification method and the finite element to Kasakura model in 3D.

References